QUIZ 4 SOLUTIONS

(LESSONS 24-32: ESTIMATING POPULATION PARAMETERS;
INTRO TO HYPOTHESIS TESTING)
MATH 119 – FALL 2019 – KUNIYUKI
100 POINTS TOTAL

No notes or books allowed. A scientific calculator is allowed. Simplify as appropriate.
You do not have to reduce fractions. For example, 10/20 does not have to be rewritten as ½.

1) (12 points). A lecture class takes an exam. We want an interval estimate for μ, the **population mean** of exam scores in the class. A random **sample** of ten exams is selected and graded. The sample mean $\bar{x}$ is 60 points. The margin of error E for a 90% confidence interval (CI) for μ is 15 points.

- a) What is a **point estimate** for the **population mean** of exam scores in the class? (2 points)

  The point estimate is the **sample mean**, $\bar{x} = 60$ points.

- b) What is the **lower limit** of the 90% CI for μ? (2 points)

  The lower limit is 45 points, because 60 points − 15 points = 45 points.

- c) What is the **upper limit** of the 90% CI for μ? (2 points)

  The upper limit is 75 points, because 60 points + 15 points = 75 points.

- d) **Write the CI** in terms of the values of $\bar{x}$ and E. (2 points)

$$\mu = \bar{x} \pm E$$
$$\mu = 60 \pm 15 \quad (\text{in points})$$

- e) **Interpret** the CI. (2 points)

  We are 90% confident that this interval **contains** the population mean of exam scores in the class.

- f) Would a **95% CI** be wider or smaller than the 90% CI for μ? (2 points)

  A 95% CI would be **wider**.

2) (8 points). Let μ be the population mean height of adult Fredonian women. Based on a random sample of adult Fredonian women, we obtain (58 inches, 68 inches) as a 95% confidence interval (CI) for μ.

- a) What is the **sample mean** $\bar{x}$? (2 points)

$$\bar{x} = \frac{58+68}{2} = 63 \text{ inches}$$

- b) What is the **margin of error** E for the CI? (2 points)

  $E = 5$ inches.

$$E = 68-63 = 5 \text{ inches}$$
$$E = 63-58 = 5 \text{ inches}$$
$$E = \frac{68-58}{2} = 5 \text{ inches}$$

- c) **Write the CI** in terms of the values of $\bar{x}$ and E. (2 points)

$$\mu = \bar{x} \pm E$$
$$\mu = 63 \pm 5 \quad (\text{in inches})$$

- d) **Interpret** the CI, as in class. (2 points)

  We are 95% confident that this interval **contains** the population mean height of adult Fredonian women.

3) (2 points). What is α for a 90% CI?

$1-\alpha = 0.90$, so $\alpha = 0.10$

4) (8 points). Consider any of the t distributions.

- a) What is the **mean**? (2 points)

  The mean is 0.

- b) Is the **standard deviation** equal to 1, less than 1, or greater than 1? (2 points)

  The standard deviation is **greater than 1**.

- c) Yes or No: Is the distribution **symmetric** about its mean? (2 points)

  Yes.

- d) As the number of degrees of freedom (df) increases, what distribution will the t distributions approach? (2 points)

  They approach the standard normal (z) distribution.

5) (4 points). Consider any of the χ^2 distributions.

- a) Yes or No: Is the **mean** equal to 0? (2 points)

  No.

- b) Yes or No: Is the distribution **symmetric** about its mean? (2 points)

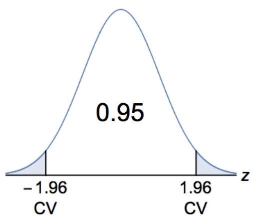
  No.

6) (2 points). In most basic applications, how many degrees of freedom (df) do we use for t and χ^2 distributions if the sample size is $n = 15$?

$\#\text{df} = n - 1 = 15 - 1 = 14$

7) (7 points). We would like to know the population mean weight of adult Fredonian men. Assume that the population standard deviation (SD) is 30 pounds, which was the sample standard deviation from a study conducted ten years ago. **Find the required sample size** n that would give us a margin of error of 5 pounds for a 95% confidence interval (CI) for the population mean. Clearly show how this is obtained by plugging into an appropriate formula.

Use these hints about the z distribution:



$$n = \left\lceil \left(\frac{z_{\alpha/2}\,\sigma}{E} \right)^2 \right\rceil$$

$$= \left\lceil \left(\frac{(1.96)(30)}{(5)} \right)^2 \right\rceil$$

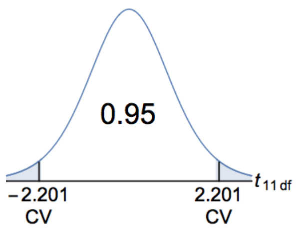
$= 139$ men (or men's weights)

8) (2 points). Interpret your answer to 7), as in class.

We need to randomly sample 139 men (or men's weights) to be 95% confident that the **eventual sample mean** will be within about 5 pounds of the population mean weight of adult Fredonian men.

9) (14 points). Banana Inc. describes "normal usage" of its cell phone, and it claims that the battery on the cell phone will last 10.00 hours under normal usage. A random sample of 12 Banana cell phone users participate in a survey and have their 12 phones monitored. Assume that Banana phone battery lifetimes are approximately normally distributed. The sample mean battery lifetime is 8.52 hours, and the sample standard deviation (SD) is 0.68 hours. You will find a 95% confidence interval (CI) for the population mean battery lifetime for the Banana cell phone.

Use these hints about the t distribution on 11 degrees of freedom (df):



- a) Why do we use a t distribution instead of a z distribution in this problem? Box in one: (2 points)

  § The population standard deviation is assumed to be unknown, and we assume that the battery lifetimes are approximately normally distributed.

  § The sample size is large, the sample mean is known, and we are estimating a population standard deviation.

- b) **Write the 95% CI** in the form $\mu = \bar{x} \pm E$. Clearly show how E is obtained by plugging into an appropriate formula. (5 points)

$$E = t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

$$\approx 2.201\left(\frac{0.68}{\sqrt{12}}\right)$$

≈ 0.43 hours, or 0.44 hours (conservative)

$\mu = \bar{x} \pm E$

$\mu \approx 8.52 \pm 0.43$ (in hours)

or 8.52 ± 0.44 (in hours) (conservative)

• c) **Write the 95% CI** in the form (lower limit, upper limit). (3 points)

$(8.09 \text{ hours},\ 8.95 \text{ hours})$, or $(8.08 \text{ hours},\ 8.96 \text{ hours})$ [conservative]

• d) **Interpret** the CI, as in class. (2 points)

We are 95% confident that this interval **contains** the population mean battery lifetime for the Banana cell phone.

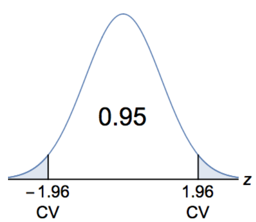
• e) H_0 states that the Banana battery lifetime is 10.00 hours. Use the significance level: $\alpha = 0.05$. Based on the 95% CI (and assuming that we are doing a two-tailed hypothesis test, as in the homework), do we decide to **reject** or **not reject** H_0? (2 points)

The 95% CI does **not** contain 10.00 hours, so we **reject** H_0.

10) (28 points). A magician's coin is flipped 300 times. It comes up heads 195 times. You will find a 95% confidence interval (CI) for p, the probability that the coin comes up heads on a flip.

Round off values of $\hat{p}$, $\hat{q}$, and E to three decimal places.

Use these hints about the z distribution:



• a) Find the sample proportion of heads, $\hat{p}$. (3 points)

$$\hat{p} = \frac{x}{n} = \frac{195}{300} = 0.650$$

• b) Find the sample proportion of tails, $\hat{q}$. (3 points)

$$\hat{q} = 1 - \hat{p} = 1 - 0.650 = 0.350$$

• c) **Verify** that normal approximations are appropriate in this problem. (4 points)

$$n\hat{p} = (300)(0.650) = 195 \geq 5$$

(Note: $n\hat{p} = x$, the number of successes, or heads.)

$$n\hat{q} = (300)(0.350) = 105 \geq 5$$

(Note: $n\hat{q} =$ the number of failures, or tails.)

• d) **Write the 95% CI** in the form $p = \hat{p} \pm E$. Clearly show how E is obtained by plugging into an appropriate formula. (7 points)

$$\begin{aligned} E &= z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} \\ &\approx 1.96\sqrt{\frac{(0.650)(0.350)}{300}} \\ &\approx 0.054 \end{aligned}$$

$$p = \hat{p} \pm E$$
$$p = 0.650 \pm 0.054$$

• e) **Write the 95% CI** in the form (lower limit, upper limit). (3 points)

$(0.596,\ 0.704)$

• f) **Interpret** the CI. (2 points)

We are 95% confident that this interval **contains** the probability that the coin comes up heads on a flip.

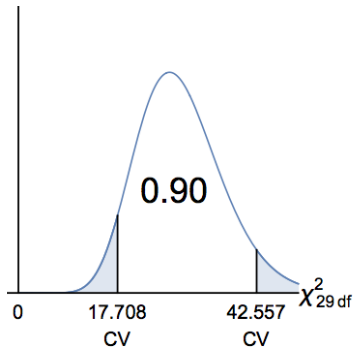
• g) We want to know the probability that the coin comes up heads on a flip. **Find the required sample size** n that would give us a margin of error of 0.03 for a 95% confidence interval (CI) for this probability. Clearly show how this is obtained by plugging into an appropriate formula (the one giving us a conservative estimate). (6 points)

$$\begin{aligned} n &= \left\lceil \frac{(z_{\alpha/2})^2(0.25)}{E^2} \right\rceil \\ &= \left\lceil \frac{(1.96)^2(0.25)}{(0.03)^2} \right\rceil \\ &= 1068 \text{ flips} \end{aligned}$$

11) (10 points). We would like to know the population standard deviation of the weights of adult Fredonian men. Assume that the weights of adult Fredonian men are approximately **normally distributed**. We randomly select 30 adult Fredonian men. The sample variance is 1005 square pounds.

- a) **Find** a 90% confidence interval (CI) for the population standard deviation (SD) of the weights of adult Fredonian men. Clearly show how the limits are obtained by plugging into an appropriate formula. **Write** the CI in either the form $\text{lower limit} < \sigma < \text{upper limit}$ or $(\text{lower limit, upper limit})$. (8 points)

Use these hints about the χ^2 distribution on 29 degrees of freedom (df):



$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(30-1)(1005)}{42.557}} < \sigma < \sqrt{\frac{(30-1)(1005)}{17.708}}$$

$$26 \text{ pounds} < \sigma < 41 \text{ pounds}$$

The **90% CI** for σ can be written as: $(26 \text{ pounds},\ 41 \text{ pounds})$

- b) **Interpret** the CI, as in class. (2 points)

We are 90% confident that this interval **contains** the population standard deviation of the weights of adult Fredonian men.

12) (3 points). A magician's coin will be flipped 100 times. H_0 states that the coin is **fair**. H_1 states that the coin is **not fair**. Let's say we observe 60 heads among the 100 flips. Let X = the number of heads in 100 flips of a fair coin. Use the significance level: $\alpha = 0.01$.

Using a two-tailed P-value analysis,

$$\begin{aligned}\text{Two-tailed } P\text{-value} &= P(X \geq 60 \text{ or } X \leq 40) \\ &\approx 0.0569, \text{ or } 5.69\%\end{aligned}$$

Based on this two-tailed P-value, do we decide to **reject** or **not reject** H_0?

We do **not** reject H_0, because P-value > α. The P-value is **not** low.