QUIZ 4 (CHAPTER 7) - SOLUTIONS

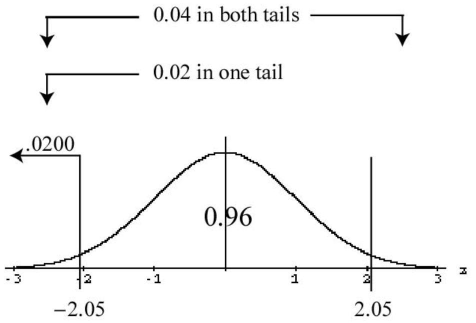
MATH 119 – SPRING 2013 – KUNIYUKI
105 POINTS TOTAL, BUT 100 POINTS = 100%

1) We want to conduct a study to estimate the mean I.Q. of a pop singer's fans. We want to have 96% confidence that the resulting sample mean will be within 2.5 I.Q. points of the true population mean for all of the singer's fans. Assuming that the population standard deviation is 15 points (which is the standard deviation for all Americans; we are probably being conservative), how large should the sample be? Use the formula:

$$n = \left\lceil \left(\frac{z_{\alpha/2}\sigma}{E} \right)^2 \right\rceil$$

(8 points)

Find the positive critical value (CV), $z_{\alpha/2}$, for a 96% confidence interval. It is about 2.05 (actually, between 2.05 and 2.06). In fact, 2.06 is the more conservative choice.



$$n = \left\lceil \left(\frac{(2.05)(15)}{2.5} \right)^2 \right\rceil = \boxed{152 \text{ [fans]}}$$

, or, conservatively,

$$n = \left\lceil \left(\frac{(2.06)(15)}{2.5} \right)^2 \right\rceil = \boxed{153 \text{ [fans]}}$$

2) The students in a huge university lecture class take a test. Assume that their test scores will be approximately normally distributed. After the students leave the lecture hall, a teaching assistant randomly samples eight tests and grades them. The sample mean is 61.5 points, and the sample standard deviation is 3.7 points. (43 points total for a)-d) below)

a) Give a 95% confidence interval for the mean test score for the entire class after all the exams are graded. Use the *t* table. (18 points)

We use the *t* table, because:

- We are estimating a population mean (μ).
- The population standard deviation (σ) is unknown.
- The distribution of test scores is approximately normal, so the CLT applies.

Find the positive critical value (CV), $t_{\alpha/2}$, for a 95% confidence interval.
We need to have $1-0.95=0.05$ total area (or probability) in the two tails.

We need to use a *t* distribution on $n-1=8-1=7$ degrees of freedom (df).
We want the 0.05 (two tails) column. It turns out $t_{\alpha/2}$ is about 2.365.

A 95% confidence interval for μ, the [population] mean test score for the class:

$$\mu=\bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

$$\mu=61.5 \pm 2.365\left(\frac{3.7}{\sqrt{8}}\right)$$

$$\mu=61.5 \pm 3.1$$

(Rounding “off” and rounding “up” the margin of error yield the same results.)

$$\boxed{58.4 \text{ points} < \mu < 64.6 \text{ points}}$$

b) Interpret your answer in part a), as we have done in class. (5 points)

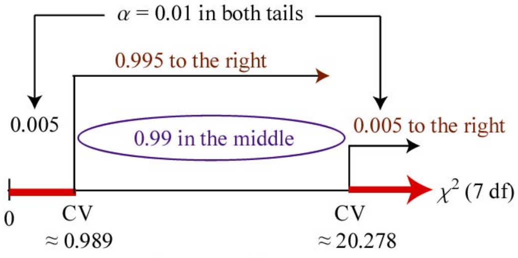
We are 95% confident that this confidence interval contains μ, the [population] mean test score for the entire class.

c) Give a 99% confidence interval for the population standard deviation of the test scores for the entire class. Use the formula:

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}}<\sigma<\sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

(15 points)

(The distribution of test scores is approximately normal, so we may proceed.)
Find the critical values (CVs), χ_R^2 and χ_L^2, for a 99% confidence interval.
We need to use the χ^2 distribution on $n-1=8-1=7$ degrees of freedom (df).
It turns out χ_R^2 is about 20.278, and χ_L^2 is about 0.989.



(not to scale)

A 99% confidence interval for σ, the [population] standard deviation of the test scores for the entire class:

$$\sqrt{\frac{(7)(3.7)^2}{20.278}}<\sigma<\sqrt{\frac{(7)(3.7)^2}{0.989}}$$

Rounding “off”: $\boxed{2.2 \text{ points} < \sigma < 9.8 \text{ points}}$

Rounding “out”: $\boxed{2.1 \text{ points} < \sigma < 9.9 \text{ points}}$

d) Interpret your answer in part c), as we have done in class. (5 points)

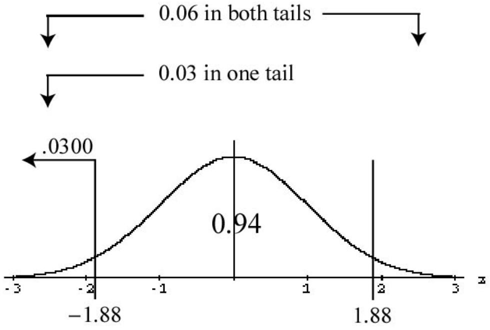
We are 99% confident that this confidence interval contains σ,
the [population] standard deviation of the test scores for the entire class.

3) Body temperatures are approximately normally distributed with mean $98.20^\circ F$ and standard deviation $0.62^\circ F$. Some anthropologists are studying the body temperatures of people living on a large island. We assume that the body temperatures of these islanders have a standard deviation of $0.62^\circ F$ (just as before; this may be conservative). We randomly sample 70 of these islanders and take their body temperature, and their sample mean body temperature is $98.50^\circ F$. Give a 94% confidence interval for the population mean body temperature of the islanders. Use the *z* table. (18 points)

We use the *z* table, because:

- We are estimating a population mean (μ).
- The population standard deviation (σ) is presumed known.
- The sample size (*n*) is 70, which is larger than 30, so the CLT applies.

Find the positive critical value (CV), $z_{\alpha/2}$, for a 94% confidence interval. It is about 1.88 (actually, between 1.88 and 1.89). In fact, 1.89 is the more conservative choice.



94% confidence interval for μ, the [population] mean body temperature of the islanders:

$$\mu = \bar{x} \pm z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$\mu = 98.50 \pm 1.88\left(\frac{0.62}{\sqrt{70}}\right)$$

$$\mu = 98.50 \pm 0.14$$

$$\boxed{98.36^o F < \mu < 98.64^o F}$$

If we use $z_{\alpha/2} = 1.89$ and round up the margin of error to be conservative, we obtain:

$$\mu = 98.50 \pm 0.15$$

$$\boxed{98.35^o F < \mu < 98.65^o F}$$

4) We are conducting a poll using a random sample of 950 American adults. Among these selected American adults, 600 believe that military service is more stressful than most other jobs. Find a 95% confidence interval for the true proportion of American adults who believe that military service is more stressful than most other jobs. Your sample proportion and limits must be rounded to four decimal places of accuracy. You may assume that the normal approximation to the binomial distribution can be applied without showing that. Do <u>not</u> use continuity corrections. Use the z table. Use the formula:

$$p = \hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

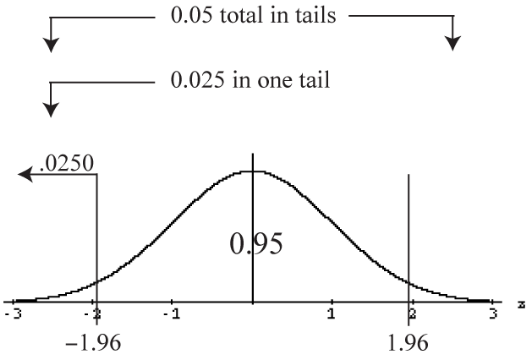
(20 points)

Let p represent the true proportion of American adults who believe that military service is more stressful than most other jobs.

The sample proportion $\hat{p} = \frac{x}{n} = \frac{600}{950} \approx 0.6316$. Then, $\hat{q} = 1 - \hat{p} \approx 1 - 0.6316 \approx 0.3684$.

<u>Note</u>: $n\hat{p} \geq 5$ and $n\hat{q} \geq 5$, which suggests that the normal approximation to the binomial distribution can be applied. We can use the z table.

Find the positive critical value (CV), $z_{\alpha/2}$, for a 95% confidence interval. It is about 1.96.



A 95% confidence interval for p:

$$p = \hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$p = 0.6316 \pm 1.96\sqrt{\frac{(0.6316)(0.3684)}{950}}$$

$$p = 0.6316 \pm 0.0307$$

$$\boxed{0.6009 < p < 0.6623}$$

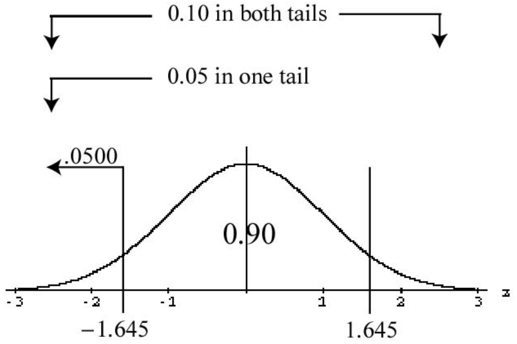
Note 1: Rounding “off” and rounding “up” the margin of error yield the same results.
Note 2: A Rasmussen poll a few years ago indicated that 66% of Americans said so.

5) We want to know the proportion of *South Park* viewers who would like to see Kenny die in the next episode. We will take a random sample of *South Park* viewers to participate in our survey. We want to have 90% confidence that the resulting sample proportion will be no more than 5% away from the true proportion for the population. How large should the sample be? Be conservative; we have no idea what the sample proportion will be. Use the formula:

$$n = \left\lceil \frac{\left(z_{\alpha/2}\right)^2 \hat{p}\hat{q}}{E^2} \right\rceil$$

(10 points)

The positive critical value (CV), $z_{\alpha/2}$, for a 90% confidence interval is about 1.645.



(Be conservative: let $\hat{p} = \hat{q} = 0.5$.) $n = \left\lceil \frac{(1.645)^2 \overbrace{(0.5)(0.5)}^{=0.25}}{(0.05)^2} \right\rceil = \boxed{271\ [\textit{South Park}\ \text{viewers}]}$

6) Results from a sample are used to construct both 95% and 99% confidence intervals for a population mean of women's heights in a country. We assume that those heights are approximately normally distributed. Assume that the intervals are constructed using the methods we have used in class.
(6 points total)

a) Which of the following is true? Box in one: (3 points)

i. The center of the 99% confidence interval is less than the center of the 95% confidence interval.

ii. The center of the 99% confidence interval is greater than the center of the 95% confidence interval.

[✓ iii. The center of the 99% confidence interval is the same as the center of the 95% confidence interval.]

The center of both confidence intervals is the sample mean.

b) Which of the following is true? Box in one: (3 points)

i. The margin of error of the 99% confidence interval is less than the margin of error of the 95% confidence interval.

[✓ ii. The margin of error of the 99% confidence interval is greater than the margin of error of the 95% confidence interval.]

iii. The margin of error of the 99% confidence interval is the same as the margin of error of the 95% confidence interval.

The 99% confidence interval is wider than the 95% confidence interval.