QUIZ 4 SOLUTIONS

(LESSONS 24-32: ESTIMATING POPULATION PARAMETERS)
MATH 119 – SPRING 2024 – KUNIYUKI
100 POINTS TOTAL

No notes or books allowed. A scientific calculator is allowed. Simplify as appropriate.
You do not have to reduce fractions. For example, 10/20 does not have to be rewritten as ½.

1) (12 points). We want to know the average number of units taken by students at a college this term. We randomly sample 70 students at the college this term. The sample mean $\bar{x}$ is 10.8 units. For a 90% confidence interval (CI) for μ, the population mean number of units taken by students at the college this term, the margin of error E is found to be 0.6 units.

- a) What is a **point estimate** for the **population mean** number of units taken by students at the college this term? (2 points)

    The point estimate is the **sample mean**, $\bar{x}$, which is 10.8 units.

- b) What is the **lower limit** of the 90% CI for μ? (2 points)

    The lower limit is 10.2 units, because $10.8 - 0.6 = 10.2$ units.

- c) What is the **upper limit** of the 90% CI for μ? (2 points)

    The upper limit is 11.4 units, because $10.8 + 0.6 = 11.4$ units.

- d) **Write the 90% CI** for μ in terms of the values of $\bar{x}$ and E. (2 points)

$$\mu = \bar{x} \pm E$$
$$\mu = 10.8 \pm 0.6 \text{ (in units)}$$

- e) **Interpret** the **90% CI** for μ, as in class. (2 points)

    We are 90% confident that this interval **contains** the population mean number of units taken by students at the college this term.

- f) Based on the same sample, would a **99% CI** be wider or smaller than the 90% CI for μ? (2 points)

    A 99% CI would be **wider**.

2) (8 points). A random sample of 45 quarters in circulation in the U.S. are weighed. A bank wants to know μ, the population mean weight (in grams) of all quarters in circulation in the U.S. We obtain $(5.604 \text{ grams}, 5.636 \text{ grams})$ as a 95% confidence interval (CI) for μ.

- a) What is the **sample mean** $\bar{x}$? (2 points)

$$\bar{x} = \frac{5.604 + 5.636}{2} = 5.620 \text{ grams}$$

- b) What is the **margin of error** E for the 95% CI for μ? (2 points)

$E = 0.016$ grams.

$$E = 5.636 - 5.620 = 0.016 \text{ grams}$$

$$E = 5.620 - 5.604 = 0.016 \text{ grams}$$

$$E = \frac{5.636 - 5.604}{2} = 0.016 \text{ grams}$$

- c) **Write the 95% CI for** μ in terms of the values of $\bar{x}$ and E. (2 points)

$$\mu = \bar{x} \pm E$$

$$\mu = 5.620 \pm 0.016 \quad \text{(in grams)}$$

- d) **Interpret** the **95% CI** for μ, as in class. (2 points)

We are 95% confident that this interval **contains** the population mean weight (in grams) of all quarters in circulation in the U.S.

3) (2 points). What is α for an 80% CI?

$1 - \alpha = 0.80$, so $\alpha = 0.20$

4) (8 points). Consider any of the t distributions.

- a) What is the **mean**? (2 points)

The mean is 0.

- b) Is the **standard deviation** equal to 1, less than 1, or greater than 1? (2 points)

The standard deviation is **greater than 1**.

- c) Yes or No: Is the distribution **symmetric** about its mean? (2 points)

Yes.

- d) As the number of degrees of freedom (df) increases, what distribution will the t distributions approach? (2 points)

They approach the standard normal (z) distribution.

5) (8 points). Consider the χ^2 distributions.

- a) Yes or No: Consider any of the χ^2 distributions.
  Is the **mean** equal to 0? (2 points)

  No.

- b) Yes or No: Consider any of the χ^2 distributions.
  Is the distribution **symmetric** about its mean? (2 points)

  No.

- c) Yes or No: As the number of degrees of freedom gets larger and approaches infinity, do the χ^2 distributions approach **a normal bell shape**? (2 points)

  Yes.

- d) Yes or No: As the number of degrees of freedom gets larger and approaches infinity, do the χ^2 distributions approach **the standard normal distribution**? (2 points)

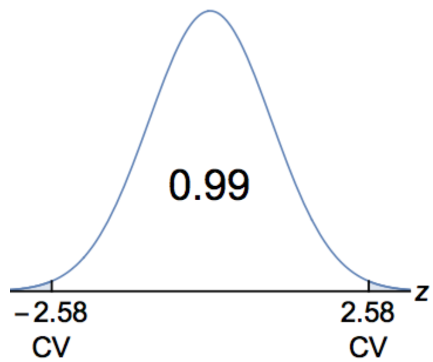
  No.

6) (2 points). In most basic applications, how many degrees of freedom (df) do we use for t and χ^2 distributions if the sample size is 90?

$$\#\text{df} = n - 1 = 90 - 1 = 89$$

7) (7 points). The college in Problem 1) defines a "working student" as a student who works a total of at least 20 hours per week at one or more jobs. The college would like to know μ, the population mean number of units taken by its working students this term. **Find the required sample size** n that would give us a margin of error of 0.5 units for a 99% confidence interval (CI) for the population mean. Assume that the population standard deviation (SD) is 3.1 units, which was the sample standard deviation from the study in Problem 1); this will likely give us a conservative estimate. Clearly show how this is obtained by plugging into an appropriate formula.

Use these hints about the z distribution:



$$n = \left\lceil \left(\frac{z_{\alpha/2}\,\sigma}{E} \right)^2 \right\rceil$$

$$= \left\lceil \left(\frac{(2.58)\,(3.1)}{(0.5)} \right)^2 \right\rceil$$

$$= 256 \text{ working students}$$

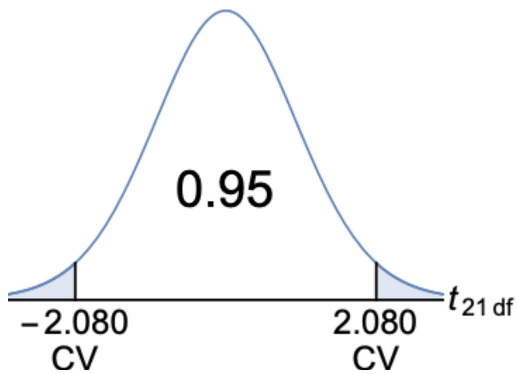
8) (2 points). In Problem 7), we want a higher confidence level and a lower margin of error compared to Problem 1), but we are basically assuming the same variation as in Problem 1). Box in the correct answer:

• The correct minimum required sample size from Problem 7) is **more than** 70 students, the sample size from Problem 1).

• The correct minimum required sample size from Problem 7) is **fewer than** 70 students, the sample size from Problem 1).

9) (14 points). A pharmaceutical company claims that its blood pressure pill lowers systolic blood pressure in American adults. A random sample of 22 American adults participate in a study on the pill. The drops in systolic blood pressure in the sample have mean 5.32 mm Hg and standard deviation (SD) 3.11 mm Hg. Assume that drops in systolic blood pressure among American adults who use the pill are approximately normally distributed. You will find a 95% confidence interval (CI) for μ, the population mean drop in systolic blood pressure among American adults taking the pill.

Use these hints about the *t* distribution on 21 degrees of freedom (df):



- a) We are estimating a population mean, and we assume that drops in systolic blood pressure among American adults who use the pill are approximately normally distributed (so the Central Limit Theorem applies here). Why do we use a *t* distribution instead of a *z* distribution in this problem? Box in one: (2 points)

  $\boxed{\text{§ The population standard deviation is assumed to be unknown.}}$

  § The population standard deviation is assumed to be known.

- b) **Write the 95% CI** for μ in the form $\mu = \bar{x} \pm E$. Clearly show how E is obtained by plugging into an appropriate formula and round E to two decimal places. (7 points)

$$\begin{aligned} E &= t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) \\ &\approx 2.080\left(\frac{3.11}{\sqrt{22}}\right) \\ &\approx 1.38 \text{ [mm Hg]} \end{aligned}$$

$$\begin{aligned} \mu &= \bar{x} \pm E \\ \mu &\approx 5.32 \pm 1.38 \text{ (in mm Hg)} \end{aligned}$$

• c) **Write the 95% CI** for μ in the form $(\text{lower limit, upper limit})$. (3 points)

$(3.94 \text{ mm Hg}, 6.70 \text{ mm Hg})$

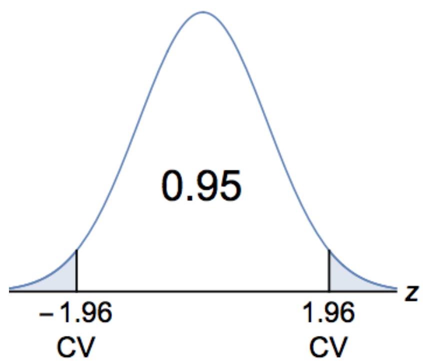
• d) **Interpret** the CI from b) and c), as in class. (2 points)

We are 95% confident that this interval **contains** the population mean drop in systolic blood pressure among American adults who use the pill.

10) (27 points). A magician's coin is flipped 700 times. It comes up heads 383 times. You will find a 95% confidence interval (CI) for p, the probability that the coin comes up heads on a flip.

Round off values of $\hat{p}$, $\hat{q}$, and E to three decimal places.

Use these hints about the z distribution:



• a) Find the sample proportion of heads, $\hat{p}$. (2 points)

$$\hat{p} = \frac{x}{n} = \frac{383}{700} = 0.547$$

• b) Find the sample proportion of tails, $\hat{q}$. (2 points)

$$\hat{q} = 1 - \hat{p} = 1 - 0.547 = 0.453$$

• c) **Verify** that normal approximations are appropriate in this problem, as in class. (4 points)

$$n\hat{p} = (700)(0.547) = 383 \geq 5$$

(Note: $n\hat{p} = x$, the number of successes, or heads.)

$$n\hat{q} = (700)(0.453) = 317 \geq 5$$

(Note: $n\hat{q}$ = the number of failures, or tails.)

• d) **Write the 95% CI** for p in the form $p = \hat{p} \pm E$. Clearly show how E is obtained by plugging into an appropriate formula. (8 points)

$$
\begin{aligned}
E &= z_{\alpha/2}\sqrt{\frac{\widehat{p}\widehat{q}}{n}} \\
&\approx 1.96\sqrt{\frac{(0.547)(0.453)}{700}} \\
&\approx 0.037
\end{aligned}
$$

$$
\begin{aligned}
p &= \widehat{p} \pm E \\
p &= 0.547 \pm 0.037
\end{aligned}
$$

• e) **Write the 95% CI** for p in the form $(\text{lower limit, upper limit})$. (3 points)

$(0.510, 0.584)$

• f) **Interpret** the CI from d) and e), as in class. (2 points)

We are 95% confident that this interval **contains** the probability that the coin comes up heads on a flip.

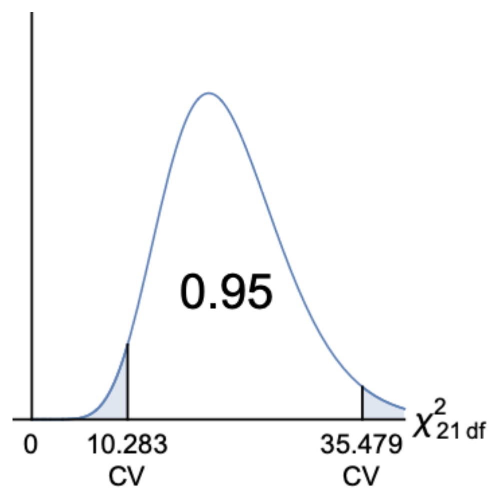
• g) We want to know the probability that the coin comes up heads on a flip. **Find the required sample size** n that would give us a margin of error of 0.01 for a 95% confidence interval (CI) for this probability. Clearly show how this is obtained by plugging into an appropriate formula (the one giving us a conservative estimate). (6 points)

$$
\begin{aligned}
n &= \left\lceil \frac{(z_{\alpha/2})^2(0.25)}{E^2} \right\rceil \\
&= \left\lceil \frac{(1.96)^2(0.25)}{(0.01)^2} \right\rceil \\
&= 9604 \text{ flips}
\end{aligned}
$$

11) (10 points). The pharmaceutical company from Problem 9) also wanted to know the population standard deviation of the drops in systolic blood pressure among American adults who use its pill. Assume that such drops are approximately normally distributed. A random sample of 22 American adults who take the pill has a standard deviation (SD) of 3.11 mm Hg.

- a) **Find** a 95% confidence interval (CI) for σ, the population standard deviation (SD) of the drops in systolic blood pressure among American adults who use the pill. Clearly show how the limits are obtained by plugging into an appropriate formula. **Write** the CI in either the form $\text{lower limit} < \sigma < \text{upper limit}$ or $(\text{lower limit, upper limit})$. Round the limits to two decimal places. (8 points)

Use these hints about the χ^2 distribution on 21 degrees of freedom (df):



$$\sqrt{\frac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}$$

$$\sqrt{\frac{(22-1)(3.11)^2}{35.479}} < \sigma < \sqrt{\frac{(22-1)(3.11)^2}{10.283}}$$

$$2.39 \text{ mm Hg} < \sigma < 4.44 \text{ mm Hg}, \text{ or}$$
$$2.39 \text{ mm Hg} < \sigma < 4.45 \text{ mm Hg} \text{ [conservative; "rounding out"]}$$

The **95% CI** for σ can be written as:

$$(2.39 \text{ mm Hg}, \ 4.44 \text{ mm Hg}), \text{ or}$$
$$(2.39 \text{ mm Hg}, \ 4.45 \text{ mm Hg}) \text{ [conservative; "rounding out"]}$$

- b) **Interpret** the CI, as in class. (2 points)

We are 95% confident that this interval **contains** the population standard deviation of the drops in systolic blood pressure among American adults who use the pill.