SOLUTIONS TO THE FINAL

(LESSONS 33-43: HYPOTHESIS TESTING; CORRELATION AND REGRESSION)
MATH 119 – FALL 2019 – KUNIYUKI
120 POINTS TOTAL

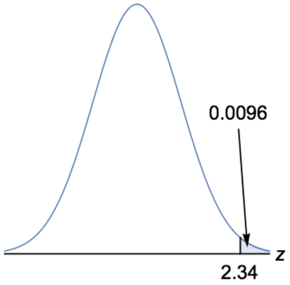
No notes or books allowed. A scientific calculator is allowed. Simplify as appropriate.
You do not have to reduce fractions. For example, 10/20 does not have to be rewritten as ½.

If you round off in the middle, round off to at least five significant digits.

1) (28 points). A particular governor wants to know if a majority of likely voters approve of the governor. (A majority means more than 50%.) Test the claim that a majority of likely voters approve of the governor at the 0.05 significance level. Warning: Write percents as decimals.

Let p = the proportion of likely voters who approve of the governor.

Use these hints about the z distribution:



- a) Write the **setup** for the hypothesis test. The setup will include H_0, H_1, identifying which is the claim, and the significance level. (5 points)

H_0: $p = 0.5$, or $p \le 0.5$
H_1: $p > 0.5$ (Claim)
$\alpha = 0.05$

- b) Is this test **two-tailed, right-tailed, or left-tailed**? (2 points)

The test is **right-tailed**.

We gather **sample data**. We randomly select 1000 likely voters in a poll. 537 of them approve of the governor.

- c) Find the **sample proportion** $\hat{p}$. (3 points)

$$\hat{p} = \frac{x}{n} = \frac{537}{1000} = 0.5370$$

- d) **Verify** that normal approximations are appropriate in this problem. (2 points)

Under H_0, $np = (1000)(0.5) = 500 \ge 5$
Under H_0, $nq = (1000)(0.5) = 500 \ge 5$
$(\text{Note: } q = 1 - p = 1 - 0.5 = 0.5)$

- e) Compute the z **test statistic** for our sample and round it off to two decimal places. Show all work, as in class! (7 points)

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.5370 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{1000}}} \approx 2.34$$

- f) Find the corresponding ***P*-value**. (2 points)

Right-tailed P-value ≈ 0.0096

- g) **Decide** whether or not to reject H_0. (2 points)

Reject H_0. (Note that $0.0096 < 0.05$, so the P-value is low.)

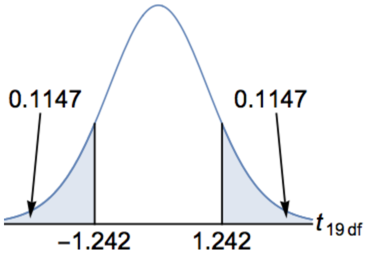
- h) Write our **conclusion** relative to the claim. (5 points)

There is **sufficient** evidence **for** the claim that a majority of likely voters approve of the governor.

2) (23 points). A pharmaceutical company makes a pill that is supposed to be 500 micrograms (mcg) by mass. The company claims that the average mass of its pills is 500 mcg. Test this claim at the 0.10 significance level. Assume that the pills (by mass) are approximately normally distributed.

Let μ = the mean mass of pills produced by the company.

Use these hints about the *t* distribution on 19 degrees of freedom (df):



- a) Write the **setup** for the hypothesis test. The setup will include H_0, H_1, identifying which is the claim, and the significance level. (5 points)

H_0: $\mu = 500$ mcg (Claim)
H_1: $\mu \neq 500$ mcg
$\alpha = 0.10$

- b) Is this test **two-tailed, right-tailed, or left-tailed**? (2 points)

The test is **two-tailed**.

We gather **sample data**. We randomly select 20 pills produced by the company. The sample mean pill mass is 495 mcg and the sample standard deviation (SD) is 18 mcg.

- c) Compute the *t* **test statistic** for our sample and round it off to three decimal places. Show all work, as in class! (6 points)

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{495-500}{\frac{18}{\sqrt{20}}} \approx -1.242$$

- d) Find the corresponding ***P*-value**. (3 points)

Two-tailed *P*-value $\approx 2(0.1147) \approx 0.2294$

- e) **Decide** whether or not to reject H_0. (2 points)

Do not reject H_0. (Note that 0.2294 > 0.10, so the *P*-value is not low.)

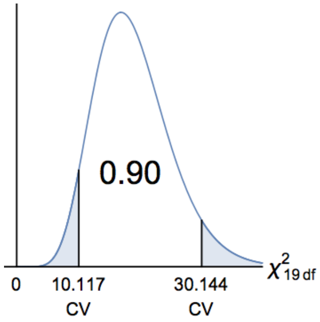
- f) Write our **conclusion** relative to the claim. (5 points)

There is **insufficient** evidence **against** the claim that the average mass of the company's pills is 500 mcg.

3) (19 points). An article on the pharmaceutical company from 2) claims that the population standard deviation (SD) of the company's pills (by mass) is 25 mcg. Test this claim at the 0.10 significance level. Assume that the pills (by mass) are approximately normally distributed. Use **the traditional (classical) method** of hypothesis testing.

Let σ = the standard deviation (SD) of the company's pills (by mass).

Use these hints about the χ^2 distribution on 19 degrees of freedom (df):



- a) Write the **setup** for the hypothesis test. The setup will include H_0, H_1, identifying which is the claim, and the significance level. (5 points)

H_0: $\sigma = 25$ mcg (Claim)
H_1: $\sigma \neq 25$ mcg
$\alpha = 0.10$

- b) Is this test **two-tailed, right-tailed, or left-tailed**? (2 points)

The test is **two-tailed**.

We gather **sample data**. As in 2), we randomly select 20 of the company's pills. The sample standard deviation (SD) is 12.5 mcg.

- c) Compute the χ^2 **test statistic** for our sample and round it off to three decimal places. Show all work, as in class! (5 points)

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(20-1)(12.5)^2}{(25)^2} \approx 4.750$$

- d) **Decide** whether or not to reject H_0. (2 points)

Reject H_0. (Note that 4.750 is in the critical region.)

- e) Write our **conclusion** relative to the claim. (5 points)

There is **sufficient** evidence **against** the claim that the population standard deviation (SD) of the company's pills (by mass) is 25 mcg.

4) (2 points). We reject a H_0 that is true. What type of error have we made? Box in one:

$\boxed{\text{Type I error}}$ Type II error

5) (21 points). According to a 2017 Gallup Poll, 51% of registered voters in California are Democrats, 30% are Republicans, and 19% are neither. Let's assume that these proportions are correct. The student newspaper at a college wants to test the claim that the distribution of party preferences among registered voters at their college is the same as for California. Test the newspaper's claim at the 0.05 significance level.

- a) Write the **setup** for the hypothesis test. The setup will include H_0, H_1, identifying which is the claim, and the significance level. (5 points)

Let p_D = the proportion of registered voters at the college who are Democrats.

Let p_R = the proportion of registered voters at the college who are Republicans.

Let p_N = the proportion of registered voters at the college who are neither.

$$H_0 : \left\{ \begin{matrix} p_D = 0.51 \\ p_R = 0.30 \\ p_N = 0.19 \end{matrix} \right\} \text{(Claim)}$$

H_1: The distribution is different.

$\alpha = 0.05$

- b) Is this test **two-tailed, right-tailed, or left-tailed**? (2 points)

The test is **right-tailed**.

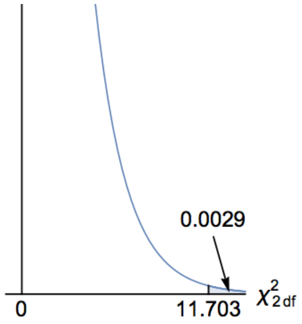
We gather **sample data**. We randomly sample 200 registered voters at the college. Among those voters, 120 are Democrats, 60 are Republicans, and 20 are neither.

- c) Write the **Observed (*O*) Table** and the **Expected (*E*) Table**. Note that each of the *E* values is at least 5, so we may apply the methods of this Lesson. (7 points)

Party preference	**Observed (*O*) Frequencies**	**Expected (*E*) Frequencies**
Democrats	120	$np_D = (200)(0.51) = 102$
Republicans	60	$np_R = (200)(0.30) = 60$
Neither	20	$np_N = (200)(0.19) = 38$

- d) The χ^2 **test statistic** is about 11.703. **Decide** whether or not to reject H_0. (2 points)

Use these hints about the χ^2 distribution on 2 degrees of freedom (df):



Reject H_0. (Note that 0.0029 < 0.05, so the *P*-value is low.)

- e) Write our **conclusion** relative to the claim. (5 points)

There is **sufficient** evidence **against** the claim that the distribution of party preferences among registered voters at their college is the same as for California.

6) (14 points). An article claims that student participation in an exam preparation program is independent of whether or not students pass the exam. Test the article's claim at the 0.05 significance level.

- a) Write the **setup** for the hypothesis test. The setup will include H_0, H_1, identifying which is the claim, and the significance level. (5 points)

  H_0: Student participation in the exam preparation program is independent of whether or not students pass the exam. (Claim)

  H_1: Student participation in the exam preparation program is dependent on whether or not students pass the exam.

  $\alpha = 0.05$

- b) Is this test **two-tailed, right-tailed, or left-tailed**? (2 points)

  The test is **right-tailed**.

We gather **sample data**, summarized in the two-way **Observed (*O*) Table** below:

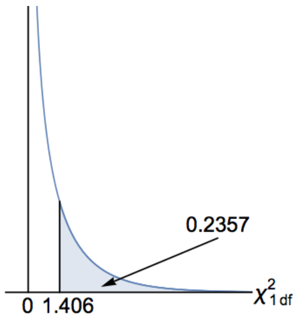
	Pass the exam	Fail the exam	**← Exam result**
Students in the program	175	125	**300**
Students not in the program	325	275	**600**
↑ Participation status	**500**	**400**	**900**

The **Expected (*E*) Table** is below:

	Pass the exam	Fail the exam	**← Exam result**
Students in the program	166.667	133.333	**300**
Students not in the program	333.333	266.667	**600**
↑ Participation status	**500**	**400**	**900**

- c) The χ^2 **test statistic** is about 1.406. We use 1 df. **Decide** whether or not to reject H_0. (2 points)

Use these hints about the χ^2 distribution on 1 degree of freedom (df):



  Do not reject H_0.

- d) Write our **conclusion** relative to the claim. (5 points)

  There is **insufficient** evidence **against** the claim that student participation in the exam preparation program is independent of whether or not students pass the exam.

7) (2 points). Fill in the blank: If a regression line for sample data is given by

$\hat{y} = 40 + 10x$, then along the regression line, for every increase of 1 unit in x,

there is an increase of 10 units in y. (2 points)

10 is the slope of the line.

8) (1 point). A student scores two standard deviations above the mean on Midterm 1 in a math class. According to the principle of **regression to the mean**, which of the following is the most likely outcome for the student on Midterm 2 in that class? Box in one:

a) The student will score three standard deviations above the mean on Midterm 2.

[✓ b) The student will score one standard deviation above the mean on Midterm 2.]

c) The student will score two standard deviations below the mean on Midterm 2.

9) (2 points). Given sample bivariate data involving two variables, x and y, we obtain $r = 0.8$ and find the corresponding least squares regression model $\hat{y} = b_0 + b_1 x$. What proportion of the variance of y is accounted for by x and the regression model? Box in the best answer below, based on the class notes and homework:

a) 8% b) 16% [✓ c) 64%] d) 80% e) 160%

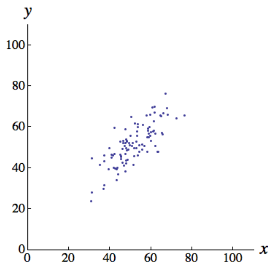
The coefficient of determination is: $r^2 = 0.64 = 64\%$.

10) (8 points). (Matching)
For each variable, the average is 50 and the standard deviation is 10.

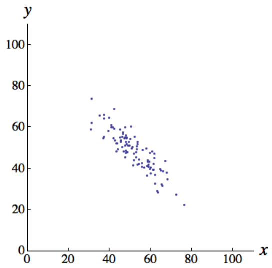
For one of the graphs below, $r = -0.90$.
For one of the graphs below, $r = 0.00$.
For one of the graphs below, $r = 0.80$.
For one of the graphs below, $r = 0.95$.

Fill in the blanks:

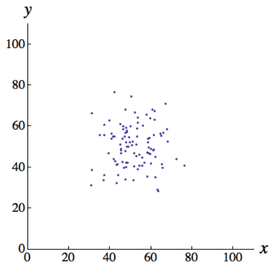
a) r for the graph below is 0.80.



b) r for the graph below is -0.90.



c) r for the graph below is 0.00.



d) r for the graph below is 0.95.