QUIZ #1 (SECTIONS 2.1, 2.2, 2.3) SOLUTIONS

MATH 121 – FALL 2003 – KUNIYUKI
105 POINTS TOTAL, BUT 100 POINTS = 100%

PART 1 (NO CALCULATORS!): 82 points

1) These instructions apply to questions a) through i):
Find the following limits without making a table. Write ∞ or $-\infty$ when appropriate. If a limit does not exist, and ∞ and $-\infty$ are inappropriate, write "DNE". **Box in your final answers.** (16 points total)

a) $\lim_{x\to 2} \frac{x-2}{x^2+2x-8}$ (6 points)

$$= \lim_{x\to 2} \frac{x-2}{(x+4)(x-2)}$$

We can cancel the $(x-2)$ factors.

$$= \lim_{x\to 2} \frac{1}{x+4}$$

Now, use direct substitution.

$$= \frac{1}{(2)+4}$$

$$= \mathbf{\frac{1}{6}}$$

b) $\lim_{x\to 2} \frac{x+3}{x-4}$ (3 points)

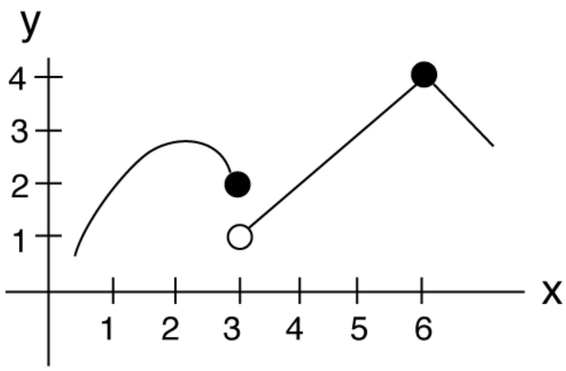
(Direct substitution works, since 2 is in the domain of this rational function.)

$$= \frac{(2)+3}{(2)-4}$$

$$= \frac{5}{-2}$$

$$= -\mathbf{\frac{5}{2}}$$

For problems c) through f), refer to the graph of f below.
Answer only is fine.



c) $\lim\limits_{x\to 3^-} f(x)$ (1 point)

= **2** (left-hand limit)

d) $\lim\limits_{x\to 3^+} f(x)$ (1 point)

= **1** (right-hand limit; it doesn't matter that the point (3,1) is deleted)

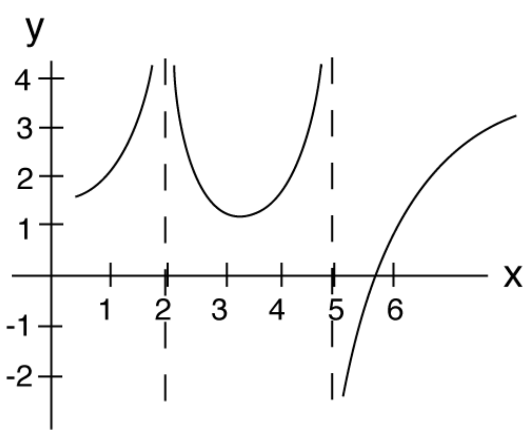
e) $\lim\limits_{x\to 3} f(x)$ (1 point)

DNE (because the left-hand and right-hand limits do not match up)

f) $\lim\limits_{x\to 6} f(x)$ (1 point)

= **4**

For problems g) through i), refer to the graph of the rational function f below.
Answer only is fine.



g) $\lim\limits_{x\to 2} f(x)$ (1 point)

$= \infty$

h) $\lim\limits_{x\to 5^+} f(x)$ (1 point)

$= -\infty$

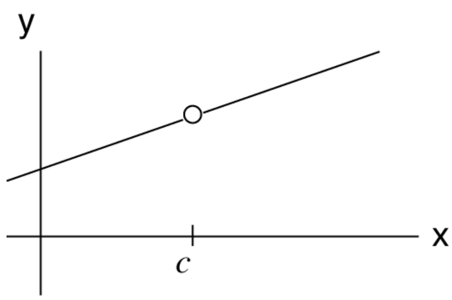
i) $\lim\limits_{x\to 5} f(x)$ (1 point)

DNE (because the left-hand and right-hand limits are mismatched)

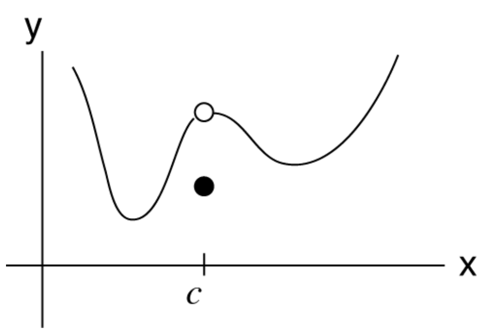
2) A function f is continuous at c if and only if the following three conditions hold:

Condition 1) $f(c)$ is defined.
Condition 2) $\lim_{x \to c} f(x)$ exists.
Condition 3) $\lim_{x \to c} f(x) = f(c)$.

In the graphs below, f is <u>not</u> continuous at c. For each graph, indicate the <u>first</u> of the above three conditions (1, 2, or 3) that fails. (9 points total; 3 points each)



1



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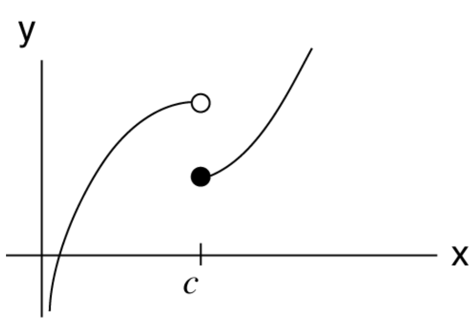


2

3) True or False: All polynomial functions of x are continuous at all real values of x. Circle one: (2 points)

True False

4) Let $f(x) = \dfrac{3x^2}{(x+9)(x-6)}$. Give all x-values where f is discontinuous. (3 points)

−9 and 6, because this rational function is continuous only on its domain.
These two values make the denominator equal to 0, so they can't be in the domain.

5) Let $f(x)=x^2-4x$. Find $f'(x)$ using the limit definition of derivative. Show all steps! (15 points)

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\left[(x+h)^2-4(x+h)\right]-\left[x^2-4x\right]}{h}$$

$$=\lim_{h\to 0}\frac{\left[x^2+2xh+h^2-4x-4h\right]-\left[x^2-4x\right]}{h}$$

$$=\lim_{h\to 0}\frac{\overbrace{x^2}^{\text{Bye!}}+2xh+h^2\overbrace{-4x}^{\text{Bye!}}-4h\overbrace{-x^2}^{\text{Bye!}}\overbrace{+4x}^{\text{Bye!}}}{h}$$

$$=\lim_{h\to 0}\frac{2xh+h^2-4h}{h}$$

$$=\lim_{h\to 0}\frac{\overbrace{h}^{\text{Bye!}}(2x+h-4)}{\underbrace{h}_{\text{Bye!}}}$$

$$=\lim_{h\to 0}\left(2x+\underbrace{h}_{\to 0}-4\right)\quad \text{(Now, you can plug in } h=0.\text{)}$$

$$=\mathbf{2x-4}$$

6) Let $f(x)=\frac{5}{x}$. Find $f'(x)$ using the limit definition of derivative. Show all steps! (15 points)

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{5}{x+h}-\frac{5}{x}}{h}$$

$$=\lim_{h\to 0}\frac{1}{h}\left(\frac{5}{x+h}-\frac{5}{x}\right)$$

LCD is $x(x+h)$.

$$=\lim_{h\to 0}\frac{1}{h}\left[\frac{5}{(x+h)}\cdot\frac{x}{x}-\frac{5}{x}\cdot\frac{(x+h)}{(x+h)}\right]$$

$$=\lim_{h\to 0}\frac{1}{h}\left[\frac{5x-5(x+h)}{x(x+h)}\right]$$

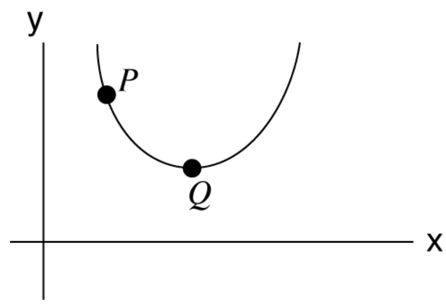
$$=\lim_{h\to 0}\frac{1}{h}\left[\frac{\overbrace{5x-5x}^{=0}-5h}{x(x+h)}\right]$$

$$=\lim_{h\to 0}\frac{1}{\underbrace{h}_{\text{Bye!}}}\left[\frac{-5\overbrace{h}^{\text{Bye!}}}{x(x+h)}\right]$$

$$=\lim_{h\to 0}\frac{-5}{x\left(x+\underbrace{h}_{\to 0}\right)}\quad \text{(Now, plug in } h=0.\text{)}$$

$$=\frac{-5}{x(x+0)}$$

$$=-\frac{\mathbf{5}}{\boldsymbol{x^2}}$$



7) What is the slope of the tangent line at the point P? Circle one: (2 points)

Positive Zero **Negative**

The tangent line at P slopes downward.

8) What is the slope of the tangent line at the point Q? Circle one: (2 points)

Positive **Zero** Negative

The tangent line at Q is horizontal.

9) If $f(x)=\frac{2}{x^7}-\sqrt[4]{x^3}+4$, find $f'(x)$. Write your answer so that it has no negative exponents. (7 points)

First, rewrite $f(x)$:

$$f(x)=2x^{-7}-x^{3/4}+4$$

$$f'(x)=-14x^{-8}-\frac{3}{4}x^{-1/4}$$

$$=-\mathbf{\frac{14}{x^8}}-\mathbf{\frac{3}{4x^{1/4}}}$$

10) If $f(x)=5x^2-4x+2$, find $f'(3)$. (4 points)

$$f'(x)=10x-4$$

$$f'(3)=10(3)-4$$

$$=\mathbf{26}$$

11) If $f(x)=\sqrt{x}$, find $\left.\frac{df}{dx}\right|_{x=9}$. (7 points)

$$f(x)=x^{1/2}$$

$$\frac{df}{dx} \text{ or } f'(x)=\frac{1}{2}x^{-1/2}$$

$$=\frac{1}{2x^{1/2}}$$

$$=\frac{1}{2\sqrt{x}}$$

Then, $\left.\frac{df}{dx}\right|_{x=9}$ or $f'(9)=\frac{1}{2\sqrt{9}}$

$$=\frac{1}{2(3)}$$

$$=\mathbf{\frac{1}{6}}$$

PART 2 (USE A SCIENTIFIC CALCULATOR!): 23 points

12) A company's profit function is given by $P(x) = 3x^2 - 4x - 400$ in dollars, where x is the number of units produced and sold. Find the marginal profit when 200 units have been produced and sold, and interpret your answer. (6 points)

$$MP(x) = P'(x) = 6x - 4$$
$$MP(200) = P'(200) = 6(200) - 4 = \mathbf{1196}$$

When 200 units have been produced and sold, the profit is increasing by about $1196 per unit (i.e., for each additional unit).

13) The number of people living on Elm Street is given by $f(t) = 1000 - 0.4t^3$, where t is measured in days $(0 \le t \le 13)$. Write units! (17 points total)

a) Find the number of people on Elm Street at $t = 5$. (3 points)

$$f(5) = 1000 - 0.4(5)^3$$
$$= \mathbf{950\ people}$$

b) Find the average rate of change of the number of people on Elm Street from $t = 3$ to $t = 8$. (8 points)

$$\frac{f(8) - f(3)}{8 - 3} = \frac{\left[1000 - 0.4(8)^3\right] - \left[1000 - 0.4(3)^3\right]}{5}$$
$$= \frac{795.2 - 989.2}{5}$$
$$= -\mathbf{38.8}\ \frac{\mathbf{people}}{\mathbf{day}}$$

(i.e., decreasing by 38.8 people per day, on average.)
That Freddy!

c) What is the instantaneous rate of change of the number of people on Elm Street at $t = 5$? (6 points)

$$f(t) = 1000 - 0.4t^3$$
$$f'(t) = -1.2t^2$$
$$f'(5) = -1.2(5)^2$$
$$= -\mathbf{30}\ \frac{\mathbf{people}}{\mathbf{day}}$$

(i.e., decreasing by 30 people per day.)