QUIZ #2 (SECTIONS 2.4, 2.5, 2.6, 2.7) SOLUTIONS

MATH 121 – FALL 2003 – KUNIYUKI
105 POINTS TOTAL, BUT 100 POINTS = 100%

PART 1 (USE A SCIENTIFIC CALCULATOR!): 31 points

1) The position function of a particle in inches is given by $s(t) = 5t^3 + 4t$, where t is time in seconds. Write units! (13 points total)

a) What is the position of the particle at time $t = 2$? (3 points)

$$\begin{aligned} s(t) &= 5t^3 + 4t \\ s(2) &= 5(2)^3 + 4(2) \\ &= 5(8) + 8 \\ &= 40 + 8 \\ &= \textbf{48 inches} \end{aligned}$$

b) What is the velocity of the particle at time $t = 2$? (5 points)

$$\begin{aligned} s(t) &= 5t^3 + 4t \\ v(t) &= s'(t) \\ &= 15t^2 + 4 \\ v(2) &= 15(2)^2 + 4 \\ &= 15(4) + 4 \\ &= 60 + 4 \\ &= \mathbf{64\ \frac{inches}{second}} \end{aligned}$$

c) What is the acceleration of the particle at time $t = 2$? (5 points)

$$\begin{aligned} v(t) &= 15t^2 + 4 \\ a(t) &= v'(t) \\ &= 30t \\ a(2) &= 30(2) \\ &= \mathbf{60\ \frac{inches}{second^2}} \quad (\textbf{or 60 inches per second per second}) \end{aligned}$$

2) A computer company's cost function is $C(x) = 400x + 5000$ in dollars, where x is the number of computers produced. (18 points total)

a) Find the average cost function, $AC(x)$.

$$\begin{aligned} AC(x) &= \frac{C(x)}{x} \\ &= \frac{400x + 5000}{x} \\ &= \frac{400x}{x} + \frac{5000}{x} \\ &= 400 + \frac{5000}{x} \quad \text{or} \quad 400 + 5000x^{-1} \end{aligned}$$

(Various forms are appropriate.)

b) Find $\lim_{x \to \infty} AC(x)$.

$$\lim_{x \to \infty} \left(400 + \underbrace{\frac{5000}{x}}_{\to 0} \right) = \mathbf{400}$$

c) Find the marginal average cost function, $MAC(x)$.

$$\begin{aligned} AC(x) &= 400 + \frac{5000}{x} \quad \text{or} \quad 400 + 5000x^{-1} \\ MAC(x) &= D_x[AC(x)] \\ &= \mathbf{-5000x^{-2}} \quad \textbf{or} \quad \mathbf{-\frac{5000}{x^2}} \end{aligned}$$

d) Evaluate $MAC(40)$ and interpret your answer.

$$\begin{aligned} MAC(40) &= -\frac{5000}{(40)^2} \\ &= \mathbf{-3.125} \end{aligned}$$

When 40 computers have been produced, the average cost is decreasing by about $3.125 (about $3.13) per computer for each additional computer produced.

PART 2 (NO CALCULATORS!): 74 points

3) Let $f(x)=\dfrac{x^2-3}{x^3+2}$. Find $f'(x)$. Simplify your answer. (10 points)

$$f'(x)=\frac{\text{Lo}\cdot\text{D(Hi)}-\text{Hi}\cdot\text{D(Lo)}}{\text{the square of what's below}} \qquad \text{(Quotient Rule)}$$

$$=\frac{(x^3+2)\cdot\left[D_x(x^2-3)\right]-(x^2-3)\cdot\left[D_x(x^3+2)\right]}{(x^3+2)^2}$$

$$=\frac{(x^3+2)\cdot(2x)-(x^2-3)\cdot(3x^2)}{(x^3+2)^2}$$

$$=\frac{2x^4+4x-(3x^4-9x^2)}{(x^3+2)^2}$$

$$=\frac{2x^4+4x-3x^4+9x^2}{(x^3+2)^2}$$

$$=\boldsymbol{\frac{-x^4+9x^2+4x}{(x^3+2)^2}}$$

You could also factor $-x$ out of the numerator, but it does not lead to further simplification in this case.

4) Find $\dfrac{d^2}{dx^2}\left(4x^3-3x^2+2\right)$. (6 points)

In other words, we want to find $f''(x)$, where $f(x)=4x^3-3x^2+2$.

$$f'(x),\text{ or } \frac{d}{dx}\left(4x^3-3x^2+2\right)=12x^2-6x$$

$$f''(x),\text{ or } \frac{d^2}{dx^2}\left(4x^3-3x^2+2\right)=\mathbf{24}\boldsymbol{x}-\mathbf{6}$$

5) For each of the following, find $f'(x)$. Simplify your answer. All exponents must be positive in your final answer. Do <u>not</u> expand out powers; for example, don't work out $(9x+4)^6$. (36 points total)

a) $f(x)=\sqrt[3]{x^2+5x}$ (8 points)

$$f(x)=\left(x^2+5x\right)^{1/3}$$

$$f'(x)=\frac{1}{3}\left(x^2+5x\right)^{-2/3}\cdot D_x\left(x^2+5x\right) \qquad \text{(by the Generalized Power Rule)}$$

$$=\frac{1}{3}\left(x^2+5x\right)^{-2/3}(2x+5)$$

$$=\mathbf{\frac{2x+5}{3\left(x^2+5x\right)^{2/3}}}$$

b) $f(x)=\dfrac{1}{(9x+4)^6}$ (8 points)

$$f(x)=(9x+4)^{-6}$$

$$f'(x)=-6(9x+4)^{-7}\cdot D_x(9x+4) \qquad \text{(by the Generalized Power Rule)}$$

$$=-6(9x+4)^{-7}\cdot(9)$$

$$=-54(9x+4)^{-7}$$

$$=-\mathbf{\frac{54}{(9x+4)^7}}$$

c) $f(x)=x^3(4x-2)^6$ (8 points)

Overall, we have a product of functions of x (that can't be combined easily), so use the Product Rule.

$$f'(x)=\left[D_x\left(x^3\right)\right]\cdot\left[(4x-2)^6\right]+\left[x^3\right]\cdot\underbrace{\left[D_x(4x-2)^6\right]}_{\text{Use the Generalized Power Rule.}}$$

$$=\left[3x^2\right]\cdot\left[(4x-2)^6\right]+\left[x^3\right]\cdot\left[6(4x-2)^5\cdot\underbrace{4}_{\text{tail}}\right]$$

Note: 4 is the tail, because it is $D_x(4x-2)$.

$$=\mathbf{3x^2(4x-2)^6+24x^3(4x-2)^5}$$

You could factor out $3x^2(4x-2)^5$.

d) $f(x)=\left(7x^2+3\right)^4(3x-10)^5$ (12 points)

Overall, we have a product of functions of x (that can't be combined easily), so use the Product Rule.

$$f'(x)=\underbrace{\left[D_x\left(7x^2+3\right)^4\right]}_{\text{Use the Generalized Power Rule.}}\cdot\left[(3x-10)^5\right]+\left[\left(7x^2+3\right)^4\right]\cdot\underbrace{\left[D_x(3x-10)^5\right]}_{\text{Use the Generalized Power Rule.}}$$

$$=\left[4\left(7x^2+3\right)^3\cdot\underbrace{14x}_{\text{tail}}\right]\cdot\left[(3x-10)^5\right]+\left[\left(7x^2+3\right)^4\right]\cdot\left[5(3x-10)^4\cdot\underbrace{3}_{\text{tail}}\right]$$

Note: $14x$ is the first tail, because it is $D_x\left(7x^2+3\right)$.

3 is the second tail, because it is $D_x(3x-10)$.

$$=\mathbf{56x\left(7x^2+3\right)^3(3x-10)^5+15\left(7x^2+3\right)^4(3x-10)^4}$$

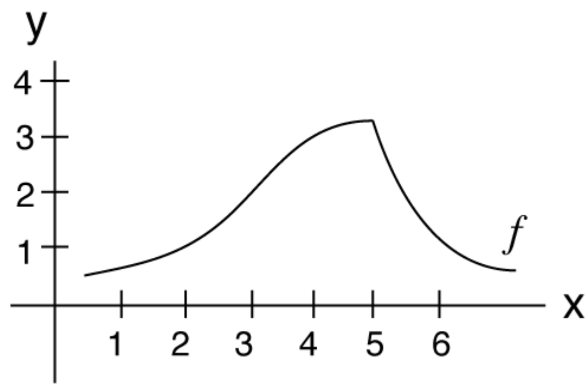
You could factor out $\left(7x^2+3\right)^3(3x-10)^4$.

6) Find functions f and g such that the function represented by $\sqrt[3]{x^2+5x}$ is the composition $f\left(g(x)\right)$. (4 points)

$\boldsymbol{g(x)}$ **or** $\boldsymbol{u=x^2+5x}$ ("Inside")

$\boldsymbol{f(u)=\sqrt[3]{u}}$

7) Consider the graph of the function f below. (18 points total; 3 points each)



For each of the following, circle one. DNE means "Does Not Exist."

a) $f'(2)$ is …

positive zero negative DNE

(f is increasing at 2.)

b) $f''(2)$ is …

positive zero negative DNE

(f is increasing at an increasing rate at 2.)

c) $f'(4)$ is …

positive zero negative DNE

(f is increasing at 4.)

d) $f''(4)$ is …

positive zero **negative** DNE

(f is increasing at a decreasing rate at 4.)

e) $f'(5)$ is …

positive zero negative **DNE**

(Left-hand and right-hand derivatives are mismatched.)

f) $f'(6)$ is …

positive zero **negative** DNE

(f is decreasing at 6.)