QUIZ #4 (SECTIONS 4.3, 4.4, 5.1, 5.2) SOLUTIONS

MATH 121 – FALL 2003 – KUNIYUKI
105 POINTS TOTAL, BUT 100 POINTS = 100%

1) Find the derivatives. Simplify where possible. (21 points total)

a) $D_x\left[\ln\left(2x^4+3\right)\right]$ (4 points)

$$= \frac{1}{2x^4+3}\cdot D_x\left(2x^4+3\right)$$
$$= \frac{1}{2x^4+3}\cdot 8x^3$$
$$= \mathbf{\frac{8x^3}{2x^4+3}}$$

b) $D_x\left(x^2\ln x^2\right)$ (8 points)

Method 1

$$= \left[D_x\left(x^2\right)\right]\left[\ln x^2\right]+\left[x^2\right]\left[D_x\underbrace{\left(\ln x^2\right)}_{\text{can use Power Rule}}\right] \quad \text{(Product Rule)}$$
$$= [2x]\left[\ln x^2\right]+\left[x^2\right]\left[D_x(2\ln x)\right]$$
$$= 2x\ln x^2+\left[x^2\right]\left[2\cdot\frac{1}{x}\right]$$
$$= \mathbf{2x\ln x^2+2x}$$

Method 2

$$= \left[D_x\left(x^2\right)\right]\left[\ln x^2\right]+\left[x^2\right]\left[D_x\left(\ln x^2\right)\right] \quad \text{(Product Rule)}$$
$$= [2x]\left[\ln x^2\right]+\left[x^2\right]\left[\frac{1}{x^2}\cdot D_x\left(x^2\right)\right]$$
$$= 2x\ln x^2+\left[x^2\right]\left[\frac{1}{x^2}\cdot 2x\right]$$
$$= \mathbf{2x\ln x^2+2x}$$

c) $D_w\left(e^{w/9}\right)$ (4 points)

$$= D_w\left(e^{\frac{1}{9}w}\right)$$
$$= \mathbf{\frac{1}{9}e^{\frac{1}{9}w}}$$

d) $D_x\left(5e^{x^2-3x}\right)$ (5 points)

$$= 5e^{x^2-3x} \cdot D_x\left(x^2-3x\right)$$
$$= \mathbf{5e^{x^2-3x}(2x-3)} \quad \textbf{or} \quad \mathbf{(10x-15)e^{x^2-3x}}$$

2) Let $f(x) = \frac{\ln x}{x^3}$. Simplify where possible: (13 points total)

a) Find $f'(x)$.

$$f'(x) = \frac{\text{Lo} \cdot \text{D(Hi)} - \text{Hi} \cdot \text{D(Lo)}}{\text{the square of what's below}} \qquad \text{(Quotient Rule)}$$
$$= \frac{\left(x^3\right) \cdot D_x(\ln x) - (\ln x) \cdot D_x\left(x^3\right)}{\left(x^3\right)^2}$$
$$= \frac{\left(x^3\right) \cdot \left(\frac{1}{x}\right) - (\ln x) \cdot \left(3x^2\right)}{\left(x^3\right)^2}$$
$$= \frac{x^2 - 3x^2 \ln x}{x^6}$$
$$= \frac{x^2(1 - 3\ln x)}{x^6}$$
$$= \mathbf{\frac{1 - 3\ln x}{x^4}} \quad \textbf{or} \quad \mathbf{\frac{1 - \ln x^3}{x^4}}$$

b) Find $f'(1)$.

$$f'(1) = \frac{1 - 3\ln 1}{(1)^4}$$
$$= \frac{1 - 3(0)}{1}$$
$$= \mathbf{1}$$

3) Find the integrals. Simplify where possible. (35 points total)

a) $\int\left(5x^2+3x-2\right)dx$ (7 points)

$$=5\left(\frac{x^3}{3}\right)+3\left(\frac{x^2}{2}\right)-2x+C$$

$$=\mathbf{\frac{5}{3}}\boldsymbol{x^3}+\mathbf{\frac{3}{2}}\boldsymbol{x^2}-\mathbf{2}\boldsymbol{x}+\boldsymbol{C}$$

b) $\int x^2(x-4)\,dx$ (7 points)

$$=\int\left(x^3-4x^2\right)dx$$

$$=\frac{x^4}{4}-4\left(\frac{x^3}{3}\right)+C$$

$$=\mathbf{\frac{1}{4}}\boldsymbol{x^4}-\mathbf{\frac{4}{3}}\boldsymbol{x^3}+\boldsymbol{C}$$

c) $\int\frac{dx}{x^8}$ (5 points)

$$=\int x^{-8}\,dx$$

$$=\frac{x^{-7}}{-7}+C$$

$$=-\mathbf{\frac{1}{7}}\boldsymbol{x^{-7}}+\boldsymbol{C}\quad\textbf{or}\quad-\frac{\mathbf{1}}{\mathbf{7}\boldsymbol{x^7}}+\boldsymbol{C}$$

d) $\int\left(\frac{2}{x}+\frac{7}{\sqrt[4]{x^3}}\right)dx$ (8 points)

$$=\int\left(2\cdot\frac{1}{x}+7x^{-3/4}\right)dx$$

$$=2\ln|x|+7\cdot\frac{x^{1/4}}{1/4}+C$$

$$=2\ln|x|+7(4)x^{1/4}+C$$

$$=\mathbf{2\ln}|\boldsymbol{x}|+\mathbf{28}\boldsymbol{x^{1/4}}+\boldsymbol{C}\quad\textbf{or}\quad\mathbf{2\ln}|\boldsymbol{x}|+\mathbf{28}\left(\sqrt[4]{\boldsymbol{x}}\right)+\boldsymbol{C}$$

e) $\int e^{0.4t}\,dt$ (4 points)

$$= \mathbf{\frac{e^{0.4t}}{0.4} + C \quad \text{or} \quad 2.5e^{0.4t} + C}$$

f) $\int \frac{7}{3x}\,dx$ (4 points)

$$= \int \frac{7}{3} \cdot \frac{1}{x}\,dx$$
$$= \mathbf{\frac{7}{3}\ln|x| + C}$$

4) A deposit of \$3400 compounded continuously at 4% interest will grow to $V(t) = 3400e^{0.04t}$ dollars after t years. Find the rate of growth after 5 years. Round off to two decimal places and write units. (7 points)

$$V'(t) = 3400(0.04)e^{0.04t}$$
$$= 136e^{0.04t}$$
$$V'(5) = 136e^{0.04(5)}$$
$$= 136e^{0.2}$$
$$\approx \mathbf{166.11\,\frac{\$}{\text{year}}} \quad \textbf{or} \quad \textbf{\$166.11 per year}$$

5) Let's say the price of an item is given by $f(t)$, where t is time in years. In class, we discussed two formulas that could be used to find the relative rate of change of the price with respect to time. Write down both formulas. (6 points)

$\frac{f'(t)}{f(t)}$ and $D_t[\ln f(t)]$.

6) A company's marginal revenue function is given by $MR(x) = 0.3e^{0.4x}$ in dollars per unit, where x is the number of units sold. (23 points total)

a) Find the revenue function. Assume that revenue is zero when nothing is sold. (10 points)

Step 1: Setup

Let $R(x)$ = the revenue when x units have been sold.

Solve $MR(x)$, or $R'(x) = 0.3e^{0.4x}$
subject to $R(0) = 0$.

Step 2: Integrate to find the general solution.

$$\int R'(x)\,dx = \int 0.3e^{0.4x}\,dx$$

$$R(x) = 0.3\left(\frac{e^{0.4x}}{0.4}\right) + C$$

$$R(x) = \frac{3}{4}e^{0.4x} + C$$

Step 3: Use the initial condition to solve for C, and find the particular solution.

When $x = 0 \Rightarrow R(x) = 0$.

$$R(x) = \frac{3}{4}e^{0.4x} + C$$

$$0 = \frac{3}{4}\overbrace{e^{0.4(0)}}^{=0} + C$$

$$0 = \frac{3}{4}(1) + C$$

$$C = -\frac{3}{4}$$

Therefore, $\boldsymbol{R(x) = \frac{3}{4}e^{0.4x} - \frac{3}{4}}$ **or** $\boldsymbol{0.75e^{0.4x} - 0.75}$

b) Use part a) to find the revenue when 7 units have been sold. Round off to the nearest cent. (3 points)

$$R(x) = 0.75e^{0.4x} - 0.75$$

$$R(7) = 0.75e^{0.4(7)} - 0.75$$

$$= 0.75e^{2.8} - 0.75$$

$$\approx \mathbf{\$11.58}$$

c) How many units have to be sold in order to achieve a revenue of \$100? Round off to the nearest integer unit. Show all steps, as we have done in class. (10 points)

Solve $R(x) = 100$ for x:

$$0.75e^{0.4x} - 0.75 = 100$$

$$0.75e^{0.4x} = 100.75$$

$$e^{0.4x} \approx 134.33$$

$$\ln e^{0.4x} \approx \ln 134.33$$

$$0.4x \approx \ln 134.33$$

$$x \approx \frac{\ln 134.33}{0.4}$$

$$x \approx \mathbf{12\ units}$$