QUIZ #1 (SAMPLE QUESTIONS)

PART 1 (NO CALCULATORS!)

1) Find the following limits without making a table. Write ∞ or $-\infty$ when appropriate. If a limit does not exist, and ∞ and $-\infty$ are inappropriate, write "DNE". **Box in your final answers.** (13 points total)

a) $\lim_{x \to -2} \frac{x}{x+1}$ (2 points)

(Direct substitution works, since -2 is in the domain of this rational function.)

$$= \frac{-2}{-2+1}$$
$$= \frac{-2}{-1}$$
$$= \mathbf{2}$$

b) $\lim_{x \to 3} \frac{x^2 - 8x + 15}{x - 3}$ (5 points)

$$= \lim_{x \to 3} \frac{(x-5)(x-3)}{(x-3)}$$

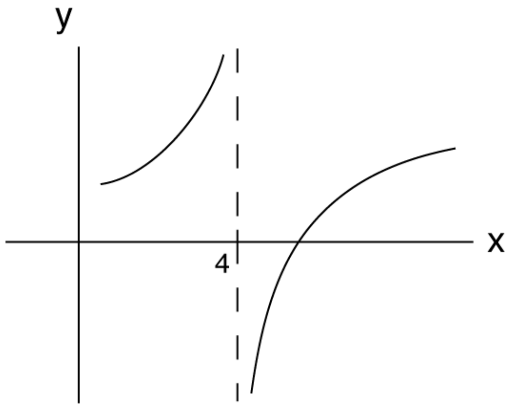
We can cancel the $(x-3)$ factors.

$$= \lim_{x \to 3} (x-5)$$

Now, use direct substitution.

$$= 3 - 5$$
$$= -\mathbf{2}$$

For problems c) through e), refer to the graph of f below.
Answer only is fine.



c) $\lim_{x \to 4^+} f(x)$ (1 point)

$= -\infty$

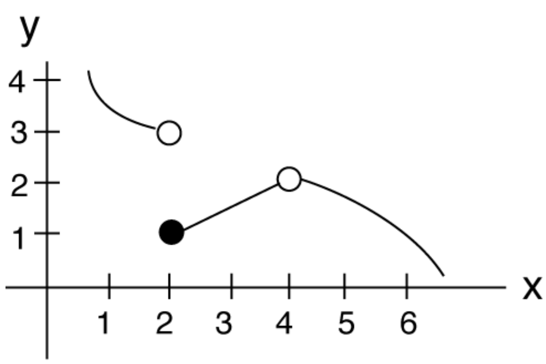
d) $\lim_{x \to 4^-} f(x)$ (1 point)

$= \infty$

e) $\lim_{x \to 4} f(x)$ (1 point)

DNE, since the answer to c), which is the right-hand "limit," and the answer to d), which is the left-hand "limit," are a mismatch.

For problems f) through h), refer to the graph of f below.
Answer only is fine.



f) $\lim_{x \to 2^-} f(x)$ (1 point)

= **3**

g) $\lim_{x \to 2} f(x)$ (1 point)

DNE, since the right-hand limit is 1, which does not equal 3, the left-hand limit.

h) $\lim_{x \to 4} f(x)$ (1 point)

= **2**. (Remember that f need not be defined at 4, itself, for the limit to exist.)

2) Let $f(x)=\dfrac{3x+2}{x^3-16x}$. Give all x-values where f is discontinuous. (5 points)

Let's factor the denominator.

$$\begin{aligned} f(x) &= \frac{3x+2}{x(x^2-16)} \\ &= \frac{3x+2}{x(x+4)(x-4)} \end{aligned}$$

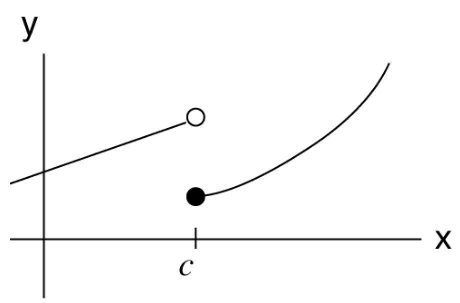
f is a rational function, so it is continuous on its natural domain. Its discontinuities are at all the x-values that are not in the natural domain (i.e., the x-values that make the denominator zero). These values are:

0, −4, and 4.

3) A function f is continuous at c if and only if the following three conditions hold:

Condition 1) $f(c)$ is defined.
Condition 2) $\lim_{x\to c} f(x)$ exists.
Condition 3) $\lim_{x\to c} f(x) = f(c)$.

In the graphs below, f is not continuous at c. For each graph, indicate the first of the above three conditions (1, 2, or 3) that fails. (6 points total; 2 points each)



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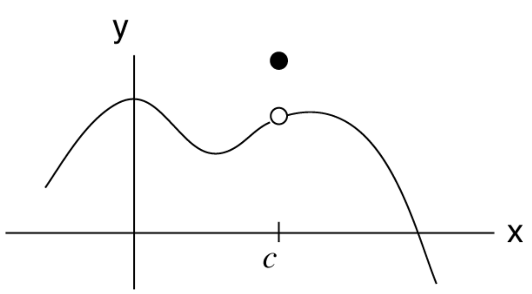

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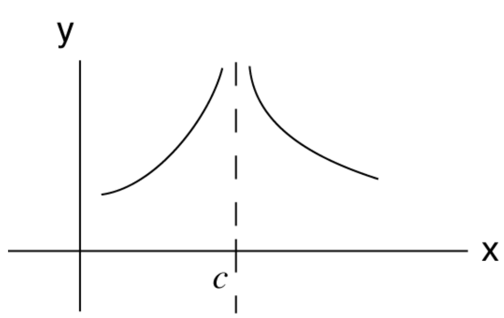

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4) Let $f(x)=4x^2+5$. Find $f'(x)$ using the limit definition of derivative. Show all steps! (10 points)

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{\left[4(x+h)^2+5\right]-\left[4x^2+5\right]}{h} \\
&= \lim_{h\to 0} \frac{\left[4\left(x^2+2xh+h^2\right)+5\right]-4x^2-5}{h} \\
&= \lim_{h\to 0} \frac{\overbrace{4x^2}^{\text{Bye!}}+8xh+4h^2\overbrace{+5}^{\text{Bye!}}\overbrace{-4x^2}^{\text{Bye!}}\overbrace{-5}^{\text{Bye!}}}{h} \\
&= \lim_{h\to 0} \frac{8xh+4h^2}{h} \\
&= \lim_{h\to 0} \frac{\overbrace{h}^{\text{Bye!}}(8x+4h)}{\underbrace{h}_{\text{Bye!}}} \\
&= \lim_{h\to 0} \left(8x+\underbrace{4h}_{\to 0}\right) \\
&= \mathbf{8}\boldsymbol{x}
\end{aligned}
$$

5) Let $f(x)=5x^4-\dfrac{1}{x^2}+\sqrt[5]{x^3}$. Find $f'(x)$. Your final answer must have only positive exponents. (6 points)

First, rewrite $f(x)$.

$$
\begin{aligned}
f(x) &= 5x^4-x^{-2}+x^{3/5} \\
f'(x) &= 20x^3+2x^{-3}+\frac{3}{5}x^{-2/5} \\
&= \mathbf{20}\boldsymbol{x^3}+\frac{\mathbf{2}}{\boldsymbol{x^3}}+\frac{\mathbf{3}}{\mathbf{5}\boldsymbol{x^{2/5}}}
\end{aligned}
$$

PART 2 (USE A SCIENTIFIC CALCULATOR!)

6) The population P of Springfield t years after January 1, 1990 $(0 \le t \le 13)$ is given by $3t^2 + 500$. You do <u>not</u> have to use the limit definition of derivative. Write units.

a) What is the average rate of change of Springfield's population between January 1, 1995 and January 1, 2000? (6 points)

January 1, 1995 corresponds to $t = 5$; January 1, 2000 corresponds to $t = 10$. Graphically, we want the slope of the secant line from $(5, P(5))$ to $(10, P(10))$.

$$\begin{aligned}\frac{P(10) - P(5)}{10 - 5} &= \frac{\left[3(10)^2 + 500\right] - \left[3(5)^2 + 500\right]}{5} \\ &= \frac{800 - 575}{5} \\ &= \frac{225}{5} \\ &= \mathbf{45}\ \frac{\textbf{people}}{\textbf{year}}\end{aligned}$$

b) What is the instantaneous rate of change of Springfield's population on January 1, 2000? (4 points)

$$\begin{aligned}P(t) &= 3t^2 + 500 \\ P'(t) &= 6t \\ P'(10) &= 6(10) \\ &= \mathbf{60}\ \frac{\textbf{people}}{\textbf{year}}\end{aligned}$$

c) What is the population of Springfield on January 1, 2000? (3 points)

$$\begin{aligned}P(t) &= 3t^2 + 500 \\ P(10) &= 3(10)^2 + 500 \\ &= \textbf{800 people}\end{aligned}$$

(You may have discovered this in part a).)