QUIZ #2 (SAMPLE QUESTIONS)

PART 1 (NO CALCULATORS!)

1) Let $f(x) = \dfrac{4x}{x^2+1}$. Find $f'(x)$. Simplify your answer. (6 points)

$$f'(x) = \frac{\text{Lo} \cdot \text{D(Hi)} - \text{Hi} \cdot \text{D(Lo)}}{\text{the square of what's below}} \qquad \text{(Quotient Rule)}$$

$$= \frac{(x^2+1) \cdot D_x(4x) - (4x) \cdot D_x(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1) \cdot (4) - (4x) \cdot (2x)}{(x^2+1)^2}$$

$$= \frac{4x^2 + 4 - 8x^2}{(x^2+1)^2}$$

$$= \mathbf{\frac{4-4x^2}{(x^2+1)^2}}$$

Note 1: Factoring the numerator does not lead to further simplification in this case.

Note 2: You could have rewritten $f(x)$ as $4x(x^2+1)^{-1}$ and then used the Product and Generalized Power Rules. However, the simplification would have been trickier.

2) Let $f(x) = x^6(3x+1)^4$. Find $f'(x)$. Simplify your answer. Do not expand out powers; for example, don't work out $(3x+1)^4$. (6 points)

First, use the Product Rule.

$$f'(x) = [x^6] \cdot \underbrace{\left[D_x(3x+1)^4\right]}_{\text{Use the Generalized Power Rule.}} + \left[D_x(x^6)\right] \cdot \left[(3x+1)^4\right]$$

$$= [x^6] \cdot \left[4(3x+1)^3 \cdot \underbrace{3}_{\text{tail}}\right] + [6x^5] \cdot \left[(3x+1)^4\right]$$

Note: 3 is the tail, because it is $D_x(3x+1)$.

$$= \mathbf{12x^6(3x+1)^3 + 6x^5(3x+1)^4}$$

3) Let $y = \sqrt[3]{x}$. Find $\left.\dfrac{d^2y}{dx^2}\right|_{x=8}$. (9 points)

In other words, find $f''(8)$, where $y = f(x) = \sqrt[3]{x}$.

$$y \text{ or } f(x) = x^{1/3}$$

$$y' \text{ or } \frac{dy}{dx} \text{ or } f'(x) = \frac{1}{3}x^{-2/3}$$

$$y'' \text{ or } \frac{d^2y}{dx^2} \text{ or } f''(x) = -\frac{2}{9}x^{-5/3}$$

$$= -\frac{2}{9x^{5/3}}$$

$$= -\frac{2}{9\left(\sqrt[3]{x}\right)^5}$$

$$f''(8) = -\frac{2}{9\left(\sqrt[3]{8}\right)^5}$$

$$= -\frac{2}{9(2)^5}$$

$$= -\frac{2}{9(32)}$$

$$= -\frac{1}{9(16)}$$

$$= -\mathbf{\frac{1}{144}}$$

4) Let $C(x)$ be a cost function, where x is the number of units produced. Find an expression for the marginal average cost function, and simplify as appropriate. Your answer will include $C(x)$ and $C'(x)$. (6 points)

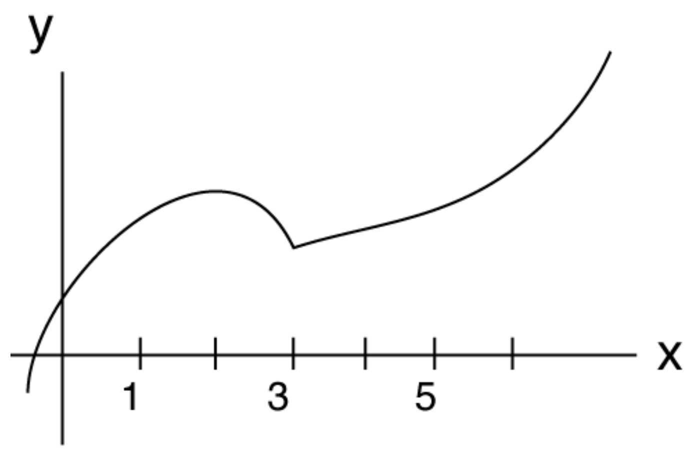
"Average" implies that we divide $C(x)$ by x.
"Marginal" implies that we then take the derivative with respect to x.

$$D_x\left[\frac{C(x)}{x}\right] = \frac{\text{Lo} \cdot \text{D(Hi)} - \text{Hi} \cdot \text{D(Lo)}}{\text{the square of what© below}} \qquad \text{(Quotient Rule)}$$

$$= \frac{x \cdot C'(x) - C(x) \cdot \overbrace{D_x(x)}^{=1}}{x^2}$$

$$= \frac{\boldsymbol{xC'(x) - C(x)}}{\boldsymbol{x^2}}$$

5) Consider the graph of the function f below. (6 points total; 2 points each)



For each of the following, indicate whether it is positive, negative, zero, or DNE (Does Not Exist).

a) $f'(3)$ ____**DNE**____ (Left-hand and right-hand derivatives are mismatched.)

b) $f'(5)$ ____**positive**____ (f is increasing at 5.)

c) $f''(1)$ ____**negative**____ (f is increasing at 1, but at a decreasing rate.)

PART 2 (USE A SCIENTIFIC CALCULATOR!)

6) The population P of Springfield t years after January 1, 1990 $(0 \le t \le 13)$ is given by $3t^2 + 500$. You do <u>not</u> have to use the limit definition of derivative. Write units.

a) What is the instantaneous rate of change of Springfield's population on January 1, 2000?

$$\begin{aligned} P(t) &= 3t^2 + 500 \\ P'(t) &= 6t \\ P'(10) &= 6(10) \\ &= \mathbf{60} \ \frac{\textbf{people}}{\textbf{year}} \end{aligned}$$

b) How fast is the rate of increase of Springfield's population changing on January 1, 2000?

$$P'(t) = 6t$$
$$P''(t) = 6$$
$$P''(10) = \mathbf{6\ \frac{people}{year^2}}\ \textbf{(i.e., people per year per year)}$$