

QUIZ #2 (SAMPLE QUESTIONS)

PART 1 (NO CALCULATORS!)

- 1) Let $f(x) = \frac{4x}{x^2 + 1}$. Find $f'(x)$. Simplify your answer. (6 points)

$$\begin{aligned} f'(x) &= \frac{\text{Lo} \cdot \text{D(Hi)} - \text{Hi} \cdot \text{D(Lo)}}{\text{the square of what's below}} && \text{(Quotient Rule)} \\ &= \frac{(x^2 + 1) \cdot D_x(4x) - (4x) \cdot D_x(x^2 + 1)}{(x^2 + 1)^2} \\ &= \frac{(x^2 + 1) \cdot (4) - (4x) \cdot (2x)}{(x^2 + 1)^2} \\ &= \frac{4x^2 + 4 - 8x^2}{(x^2 + 1)^2} \\ &= \frac{4 - 4x^2}{(x^2 + 1)^2} \end{aligned}$$

Note 1: Factoring the numerator does not lead to further simplification in this case.

Note 2: You could have rewritten $f(x)$ as $4x(x^2 + 1)^{-1}$ and then used the Product and Generalized Power Rules. However, the simplification would have been trickier.

- 2) Let $f(x) = x^6(3x + 1)^4$. Find $f'(x)$. Simplify your answer. Do not expand out powers; for example, don't work out $(3x + 1)^4$. (6 points)

First, use the Product Rule.

$$\begin{aligned} f'(x) &= [x^6] \cdot \underbrace{[D_x(3x + 1)^4]}_{\substack{\text{Use the Generalized} \\ \text{Power Rule.}}} + [D_x(x^6)] \cdot [(3x + 1)^4] \\ &= [x^6] \cdot \left[4(3x + 1)^3 \cdot \underset{\text{tail}}{3} \right] + [6x^5] \cdot [(3x + 1)^4] \end{aligned}$$

Note: 3 is the tail, because it is $D_x(3x + 1)$.

$$= 12x^6(3x + 1)^3 + 6x^5(3x + 1)^4$$

3) Let $y = \sqrt[3]{x}$. Find $\left. \frac{d^2y}{dx^2} \right|_{x=8}$. (9 points)

In other words, find $f''(8)$, where $y = f(x) = \sqrt[3]{x}$.

$$\begin{array}{rcl}
 y \text{ or } f(x) & = & x^{1/3} \\
 y' \text{ or } \frac{dy}{dx} \text{ or } f'(x) & = & \frac{1}{3}x^{-2/3} \\
 y'' \text{ or } \frac{d^2y}{dx^2} \text{ or } f''(x) & = & -\frac{2}{9}x^{-5/3} \\
 & = & -\frac{2}{9x^{5/3}} \\
 & = & -\frac{2}{9(\sqrt[3]{x})^5}
 \end{array}
 \qquad
 \begin{array}{rcl}
 f''(8) & = & -\frac{2}{9(\sqrt[3]{8})^5} \\
 & = & -\frac{2}{9(2)^5} \\
 & = & -\frac{2}{9(32)} \\
 & = & -\frac{1}{9(16)} \\
 & = & -\frac{1}{144}
 \end{array}$$

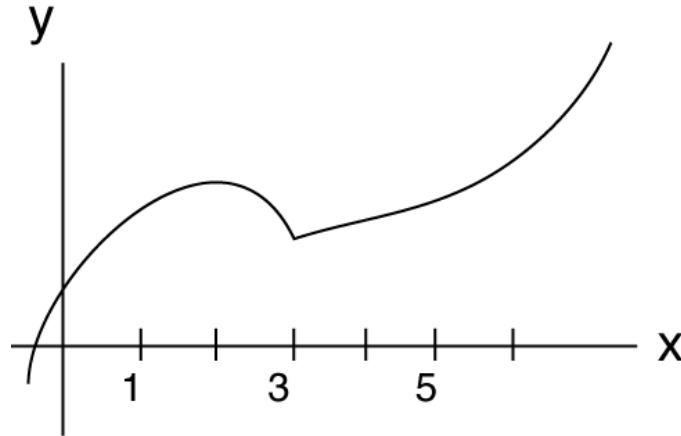
4) Let $C(x)$ be a cost function, where x is the number of units produced. Find an expression for the marginal average cost function, and simplify as appropriate. Your answer will include $C(x)$ and $C'(x)$. (6 points)

"Average" implies that we divide $C(x)$ by x .

"Marginal" implies that we then take the derivative with respect to x .

$$\begin{aligned}
 D_x \left[\frac{C(x)}{x} \right] &= \frac{\text{Lo} \cdot \text{D(Hi)} - \text{Hi} \cdot \text{D(Lo)}}{\text{the square of what's below}} && \text{(Quotient Rule)} \\
 &= \frac{x \cdot C'(x) - C(x) \cdot \overbrace{D_x(x)}^{=1}}{x^2} \\
 &= \frac{x C'(x) - C(x)}{x^2}
 \end{aligned}$$

5) Consider the graph of the function f below. (6 points total; 2 points each)



For each of the following, indicate whether it is positive, negative, zero, or DNE (Does Not Exist).

- a) $f'(3)$ DNE (Left-hand and right-hand derivatives are mismatched.)
- b) $f'(5)$ positive (f is increasing at 5.)
- c) $f''(1)$ negative (f is increasing at 1, but at a decreasing rate.)

PART 2 (USE A SCIENTIFIC CALCULATOR!)

6) The population P of Springfield t years after January 1, 1990 ($0 \leq t \leq 13$) is given by $3t^2 + 500$. You do not have to use the limit definition of derivative. Write units.

- a) What is the instantaneous rate of change of Springfield's population on January 1, 2000?

$$\begin{aligned} P(t) &= 3t^2 + 500 \\ P'(t) &= 6t \\ P'(10) &= 6(10) \\ &= 60 \frac{\text{people}}{\text{year}} \end{aligned}$$

b) How fast is the rate of increase of Springfield's population changing on January 1, 2000?

$$P'(t) = 6t$$

$$P''(t) = 6$$

$$P''(10) = 6 \frac{\text{people}}{\text{year}^2} \text{ (i.e., people per year per year)}$$