QUIZ #4 (SAMPLE SOLUTIONS)

USE A SCIENTIFIC CALCULATOR!

1) Find the derivatives. Simplify where possible. (19 points total)

a) $D_x\left[\ln\left(x^3+5\right)^4\right]$ (7 points)

Method 1

$$= D_x\left[4\ln\left(x^3+5\right)\right] \quad \text{(Using Power Rule for Logs)}$$

$$= 4\cdot\frac{1}{x^3+5}\cdot\overbrace{D_x\left(x^3+5\right)}^{=3x^2}$$

$$= \mathbf{\frac{12x^2}{x^3+5}}$$

Method 2

$$= \frac{1}{\left(x^3+5\right)^4}\cdot D_x\left[\left(x^3+5\right)^4\right]$$

$$= \frac{1}{\left(x^3+5\right)^4}\cdot 4\left(x^3+5\right)^3\left(3x^2\right)$$

$$= \frac{12x^2\left(x^3+5\right)^3}{\left(x^3+5\right)^4}$$

$$= \mathbf{\frac{12x^2}{x^3+5}}$$

b) $D_x\left(3e^{4x^2+x}\right)$ (4 points)

$$= 3e^{4x^2+x}\cdot D_x\left(4x^2+x\right)$$

$$= \mathbf{3e^{4x^2+x}(8x+1)} \quad \textbf{or} \quad \mathbf{e^{4x^2+x}(24x+3)}$$

c) $D_x\left(x^2 e^{6x}\right)$ (6 points)

$$= \left[D_x\left(x^2\right)\right]\left[e^{6x}\right] + \left[x^2\right]\left[D_x\left(e^{6x}\right)\right] \qquad \text{(Product Rule)}$$
$$= [2x]\left[e^{6x}\right] + \left[x^2\right]\left[6e^{6x}\right]$$
$$= \mathbf{2xe^{6x} + 6x^2e^{6x}}$$

d) $D_x\left(e^8\right)$ (2 points)

$$= \mathbf{0}$$ (Hey, we're differentiating a constant….)

2) If $f(x) = x\ln x$, find $f'(e)$. Do <u>not</u> approximate. (8 points)

$$f'(x) = \left[D_x(x)\right][\ln x] + [x]\left[D_x(\ln x)\right] \qquad \text{(Product Rule)}$$
$$= [1][\ln x] + [x]\left[\frac{1}{x}\right]$$
$$= \ln x + 1$$
$$f'(e) = \ln e + 1$$
$$= 1 + 1$$
$$= \mathbf{2}$$

3) Find the integrals. Simplify wherever possible. (33 points total)

a) $\int\left(4x^3 - 10x + 7\right)dx$ (6 points)

$$= 4\left(\frac{x^4}{4}\right) - 10\left(\frac{x^2}{2}\right) + 7x + C$$
$$= \boldsymbol{x^4 - 5x^2 + 7x + C}$$

<u>Note</u>: You could have used "Integrating-at-Sight" for the first term.

b) $\int\left(\sqrt[4]{x} + \frac{3}{x^4}\right)dx$ (7 points)

$$= \int\left(x^{1/4} + 3x^{-4}\right)dx$$
$$= \frac{x^{5/4}}{5/4} + 3\left(\frac{x^{-3}}{-3}\right) + C$$
$$= \boldsymbol{\frac{4}{5}x^{5/4} - x^{-3} + C}$$

c) $\int (x+3)^2\,dx$ (7 points)

$$= \int \left(x^2+6x+9\right)dx$$
$$= \frac{x^3}{3}+6\left(\frac{x^2}{2}\right)+9x+C$$
$$= \mathbf{\frac{1}{3}x^3+3x^2+9x+C}$$

d) $\int e^{-4x}\,dx$ (3 points)

$$= \frac{e^{-4x}}{-4}+C$$
$$= \mathbf{-\frac{1}{4}e^{-4x}+C}$$

e) $\int \frac{4}{5x}dx$ (4 points)

$$= \int \frac{4}{5}\cdot\frac{1}{x}dx$$
$$= \mathbf{\frac{4}{5}\ln|x|+C}$$

f) $\int \left(3+3w^{-1}+3w^{-2}\right)dw$ (6 points)

$$= 3w+3\ln|w|+3\left(\frac{w^{-1}}{-1}\right)+C$$
$$= \mathbf{3w+3\ln|w|-3w^{-1}+C}$$

4) A deposit of \$4000 compounded continuously at 3% interest will grow to $V(t)=4000e^{0.03t}$ dollars after t years. Find the rate of growth after 6 years. Round off to two decimal places and write units. (7 points)

$$V'(t)=4000(0.03)e^{0.03t}$$
$$=120e^{0.03t}$$
$$V'(6)=120e^{0.03(6)}$$
$$=120e^{0.18}$$
$$\approx \mathbf{143.67\,\frac{\$}{year}}\quad \textbf{or}\quad \textbf{\$143.67 per year}$$

5) The population of Fredonia is given by $P(t) = t^2 + 4t$, where t is the number of years from now. Find the relative rate of change of the population 10 years from now. Round off your answer to the nearest percent per year. (8 points)

Method 1

$$\frac{P'(t)}{P(t)} = \frac{2t+4}{t^2+4t}$$

At $t = 10$:

$$= \frac{2(10)+4}{(10)^2+4(10)}$$
$$= \frac{24}{140} \quad \text{or} \quad \frac{6}{35}$$
$$\approx 0.17$$

About 17% per year

Method 2

$$D_t\left[\ln P(t)\right] = D_t\left[\ln\left(t^2+4t\right)\right]$$
$$= \frac{1}{t^2+4t} \cdot D_t\left(t^2+4t\right)$$
$$= \frac{1}{t^2+4t} \cdot (2t+4)$$
$$= \frac{2t+4}{t^2+4t}$$

At $t = 10$:

$$= \frac{2(10)+4}{(10)^2+4(10)}$$
$$= \frac{24}{140} \quad \text{or} \quad \frac{6}{35}$$
$$\approx 0.17$$

About 17% per year

6) There is an epidemic of Rage virus in a city. The rate of growth of the epidemic after t days is given by $12e^{2t}$ new cases per day $(0 \le t \le 7)$. At time $t = 0$, there were 10 cases. (23 points total)

a) Find a formula for the total number of cases of Rage virus in the first t days. (10 points)

Step 1: Setup

Let $f(t)$ = the total number of cases in the first t days.

Solve $f'(t) = 12e^{2t}$
subject to $f(0) = 10$.

Step 2: Integrate to find the general solution.

$$\int f'(t)\,dt = \int 12e^{2t}\,dt$$

$$f(t) = 12\left(\frac{e^{2t}}{2}\right) + C$$

$$f(t) = 6e^{2t} + C$$

Step 3: Use the initial condition to solve for C, and find the particular solution.

When $t = 0 \Rightarrow f(t) = 10$.

$$f(t) = 6e^{2t} + C$$

$$10 = 6\underbrace{e^{2(0)}}_{\substack{=e^0 \\ =1}} + C$$

$$10 = 6 + C$$

$$C = 4$$

Therefore, $\boldsymbol{f(t) = 6e^{2t} + 4}$

b) Use part a) to find the total number of cases in the first 4 days. Round off to the nearest integer. (3 points)

$$f(4) = 6e^{2(4)} + 4$$

$$= 6e^{8} + 4$$

$$\approx \textbf{17,890 cases}$$

c) After how many days will there be 100,000 cases? Round off to the nearest tenth of a day. Show all steps, as we have done in class. (10 points)

Solve $f(t) = 100{,}000$ for t.

$$6e^{2t} + 4 = 100{,}000$$
$$6e^{2t} = 99{,}996$$
$$e^{2t} = 16{,}666$$
$$\ln e^{2t} = \ln 16{,}666$$
$$2t = \ln 16{,}666$$
$$t = \frac{\ln 16{,}666}{2}$$
$$t \approx \mathbf{4.9\ days}$$

7) True or False: If M and N are positive in value, $\ln MN = \ln M \cdot \ln N$. Circle one: (2 points)

True **False**

The log of a product = the sum of the logs:
$\ln MN = \ln M + \ln N$.