Math 141 Name: ________________________

FINAL

(CHAPTERS 7-10)
MATH 141 – FALL 2017 – KUNIYUKI
250 POINTS TOTAL

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed. A scientific calculator is allowed.

To maximize chances for partial credit, please be neat and indicate any elementary row operations (EROs) you use! Clarity is important. I might not grade "messes."

1) Find the intersection point(s) of the graphs of $3x + y = 0$ and $y^2 - x^2 = 24$ in the usual xy-plane by solving a system, as in class. Do **not** rely on graphing, "trial-and-error," guessing, or point-plotting as a basis for your method. Show all work! Write the solution set with all solutions as ordered pairs of the form (x, y). (14 points)

2) Write the PFD (Partial Fraction Decomposition) for $\dfrac{3x^3 - x^2 + 12x + 2}{\left(x^2+4\right)^2}$.

You must find the unknowns in the PFD Form. Show all work, as in class! (22 points)

3) Write the PFD (Partial Fraction Decomposition) Form for

$\dfrac{1}{x^3\left(x^2+1\right)}$. Do not find the unknowns (A, B, etc.). (7 points)

4) Let $A = \begin{bmatrix} 1 & 5 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$. (6 points)

a) What is the size of A? Box in one: 3×5 5×3

b) Is A in row-echelon form? Box in one: Yes No

c) Is A in reduced row-echelon (RRE) form? Box in one: Yes No

5) Solve the system below using matrices and Gaussian Elimination with Back-Substitution (or Gauss-Jordan Elimination, if you prefer). Write your solution as an ordered triple of the form (x, y, z) in a solution set.

Clearly indicate the elementary row operations (EROs) you are applying. Your final matrix must be in row-echelon form. (30 points)

$$\begin{cases} -3x + 14y - 8z = 42 \\ 4x - 6y - z = -1 \\ x - 4y + 3z = -13 \end{cases}$$

<u>YOU MAY CONTINUE ON THE BACK OF THE TEST.</u>

6) Let $A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$. Find the matrix $3A - B$. (8 points)

7) Let $A = \begin{bmatrix} 3 & -1 & 2 \\ 0 & 4 & 1 \\ 2 & -3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -1 \\ 2 & 0 \\ 3 & 1 \end{bmatrix}$. (15 points total)

a) Find the matrix product *AB*. (12 points)

b) The matrix product *BA* is ... (Box in one:). (3 points)

i. a matrix of size 2×3

ii. a matrix of size 3×2

iii. undefined

8) Let $A = \begin{bmatrix} 3 & 5 & 0 \\ -2 & -1 & 3 \\ 2 & 6 & 1 \end{bmatrix}$. (25 points total)

a) Find $\det(A)$ using Sarrus's Rule, the method using diagonals. (10 points)

b) Find $\det(A)$ using the Expansion by Cofactors Method. Show all work, as in class. (15 points)

9) Solve the equation $\begin{vmatrix} 5-\lambda & 1 \\ 3 & 3-\lambda \end{vmatrix} = 0$ for λ (lambda). Note: In doing so, you are finding the <u>eigenvalues</u> of the matrix $\begin{bmatrix} 5 & 1 \\ 3 & 3 \end{bmatrix}$. (7 points)

10) Simplify completely: $\frac{n!}{(n+2)!}$ $\left(n \in \mathbb{Z}^{+}\right)$. (4 points)

11) Evaluate: $\sum_{k=2}^{4}\left(2^{k}-1\right)$. (7 points)

12) Consider the sequence: $-\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \ldots.$

Write a nonrecursive expression (formula) for the apparent general n^{th} term, a_n, for this sequence. Let a_1 be the initial term; that is, assume that n begins with 1. (5 points)

13) Consider the arithmetic sequence: $8, 5, 2, -1, -4, \ldots.$ (12 points total)

a) What is d, the common difference of this sequence?

b) Write a nonrecursive expression (formula) for the general n^{th} term, a_n, for this sequence. Let a_1 be the initial term; that is, assume that n begins with 1.

c) Find a_{100}.

14) Consider the infinite geometric series: $4-4+4-4+\ldots.$ (4 points total)

a) What is r, the common ratio of this series?

b) Is this series convergent or divergent? Box in one:

Convergent Divergent

15) Consider the geometric sequence: $2,\ -\frac{1}{2},\ \frac{1}{8},\ -\frac{1}{32},\ \frac{1}{128},\ \ldots.$

(16 points total)

a) Write a nonrecursive expression (formula) for the general n^{th} term, a_n, for this sequence. Let a_1 be the initial term; that is, assume that n begins with 1.

b) Find the sum of the corresponding geometric series, $\sum_{n=1}^{\infty} a_n$, which is:

$$2-\frac{1}{2}+\frac{1}{8}-\frac{1}{32}+\frac{1}{128}-\ldots$$

c) Is the series in b) convergent or divergent? Box in one:

Convergent Divergent

16) Prove using mathematical induction: $1+2+3+\ldots+n=\frac{n(n+1)}{2}\ \left(\forall n\in\mathbb{Z}^{+}\right)$.

(14 points)

17) Use the Binomial Theorem to expand and simplify: $(x+h)^4$. (10 points)

18) Let $f(x)=x^4$. Use your answer from 17) to evaluate and simplify the following difference quotient completely: $\frac{f(x+h)-f(x)}{h}$ $(h \neq 0)$
(5 points)

19) An ellipse has equation $4x^2 + y^2 + 24x - 4y + 24 = 0$ in the usual *xy*-plane. (26 points total)

a) Find the standard form of the equation of this ellipse. (11 points)

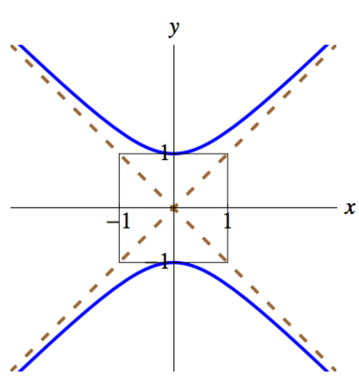
b) The center of this ellipse is at what point? (2 points)

c) The vertices of this ellipse are at what points? (4 points)

d) The foci of this ellipse are at what points? (6 points)

e) What is the eccentricity of this ellipse? (3 points)

20) The graph below is the graph of (box in one:) $x^2 - y^2 = 1$ $y^2 - x^2 = 1$ (3 points)



21) Sketch the graph of the polar equation $r = 2\cos(\theta)$, where r and θ are polar coordinates. You may use either the Cartesian or polar graph paper below; box in the one you use. Use arrows to indicate orientation. (10 points)