

FINAL**(CHAPTERS 7-10)****MATH 141 – FALL 2022 – KUNIYUKI****250 POINTS TOTAL**

Show all work, simplify as appropriate, and use “good form and procedure” (as in class).

Box in your final answers!

No notes or books allowed. A scientific calculator is allowed.

To maximize chances for partial credit, please be neat and indicate any elementary row operations (EROs) you use! Clarity is important. I might not grade “messes.”

- 1) Find the intersection point(s) of the graphs of $y = 2x^2 + 3x - 8$ and $y = x^2 + 2x - 2$ in the usual xy -plane by solving a system, as in class. Do **not** rely on graphing, “trial-and-error,” guessing, or point-plotting as a basis for your method. Show all work! Write the solution set with all solutions as ordered pairs of the form (x, y) . (12 points)

2) Write the PFD (Partial Fraction Decomposition) for $\frac{7x+17}{x^2+6x+9}$.

You must find the unknowns in the PFD Form. Show all work, as in class!
(17 points)

3) Write the PFD (Partial Fraction Decomposition) for $\frac{6x^2 - 5x + 18}{x^3 + 9x}$.

You must find the unknowns in the PFD Form. Show all work, as in class!
(20 points)

4) Write the PFD (Partial Fraction Decomposition) Form for

$\frac{1}{x^2(x-5)(x^2+9)^2}$. Do not find the unknowns (A , B , etc.). (7 points)

- 5) Solve the system below using matrices and Gaussian Elimination with Back-Substitution (or Gauss-Jordan Elimination, if you prefer). Write your solution as an ordered triple of the form (x, y, z) in a solution set. Clearly indicate the elementary row operations (EROs) you are applying. **Your final matrix must be in row-echelon form.** (30 points)

$$\begin{cases} -3x + 8y + z = -13 \\ x - 2y + 3z = -5 \\ 5x - 11y + 8z = -5 \end{cases}$$

YOU MAY CONTINUE ON THE BACK OF THE TEST.

6) Let $A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. (4 points)

a) Is A in row-echelon form? Box in one: Yes No

b) Is A in reduced row-echelon (RRE) form? Box in one: Yes No

7) Let $B = \begin{bmatrix} 1 & 0 & 5 & 4 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. (4 points)

a) Is B in row-echelon form? Box in one: Yes No

b) Is B in reduced row-echelon (RRE) form? Box in one: Yes No

8) Find the matrix product AB if: $A = \begin{bmatrix} 5 & -1 & 1 \\ -2 & 2 & 0 \\ 3 & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & 3 \\ 4 & -1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$.

(13 points)

9) Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 3 & -1 \\ 0 & 2 & 5 \end{bmatrix}$. (23 points total)

a) Find $\det(A)$ using Sarrus's Rule, the method using diagonals. (10 points)

b) Find $\det(A)$ using the Expansion by Cofactors Method.
Show all work, as in class. (13 points)

10) Simplify completely: $\frac{(3n+1)!}{(3n-1)!}$ ($n \in \mathbb{Z}^+$). (4 points)

11) Evaluate: $\sum_{i=0}^3 i!$. (7 points)

12) Consider the sequence: $1, -3, 5, -7, 9, -11, \dots$

Write a nonrecursive expression (formula) for the apparent general n^{th} term, a_n , for this sequence, as in class. Let a_1 be the initial term; that is, assume that n begins with 1. (5 points)

13) Consider the sequence defined recursively as follows.

a_1 is considered to be the first term. (7 points total)

$$\begin{cases} a_1 = 5 \\ a_{k+1} = 4a_k \quad (\forall k \in \mathbb{Z}^+) \end{cases}$$

a) Write the first four terms of the sequence. (5 points)

b) The sequence is Box in one: (2 points)

Arithmetic

Geometric

Neither

14) Consider the arithmetic sequence: $3, 10, 17, 24, 31, \dots$ (10 points total)

a) Write a nonrecursive expression (formula) for the general n^{th} term, a_n , for this sequence. Let a_1 be the initial term; that is, assume that n begins with 1.

b) Find a_{50} .

15) Consider the geometric sequence: $\frac{2}{3}, \frac{1}{2}, \frac{3}{8}, \frac{9}{32}, \frac{27}{128}, \dots$ (16 points)

a) Write a nonrecursive expression (formula) for the general n^{th} term, a_n , for this sequence. Let a_1 be the initial term; that is, assume that n begins with 1.

b) Find the sum of the corresponding geometric series, $\sum_{n=1}^{\infty} a_n$, which is:

$$\frac{2}{3} + \frac{1}{2} + \frac{3}{8} + \frac{9}{32} + \dots$$

c) Is the series in b) convergent or divergent? Box in one:

Convergent

Divergent

16) For what real values of x does the series $\sum_{n=1}^{\infty} (7x)^n$ converge? Write your

answer using interval form (the form using parentheses and/or brackets).

Note: The interval is called the “interval of convergence” for the series.

Hint: The series is geometric. (Let’s say this is true even for $x = 0$.) (4 points)

17) Prove using mathematical induction: $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$ ($\forall n \in \mathbb{Z}^+$).
(14 points)

18) Use the Binomial Theorem to expand and simplify: $(x + h)^5$. (12 points)

19) Let $f(x) = x^5$. Use your answer from 18) to evaluate and simplify the following difference quotient completely: $\frac{f(x+h) - f(x)}{h}$ ($h \neq 0$). (5 points)

20) An ellipse has equation $36x^2 + 25y^2 - 288x + 100y - 224 = 0$ in the usual xy -plane. (26 points total)

a) Find the standard form of the equation of this ellipse. (11 points)

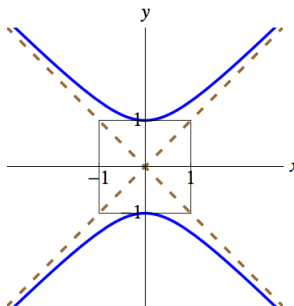
b) The center of this ellipse is at what point? (2 points)

c) The vertices of this ellipse are at what points? (4 points)

d) The foci of this ellipse are at what points? (6 points)

e) What is the eccentricity of this ellipse? (3 points)

21) The graph below is the graph of (box in one:) $x^2 - y^2 = 1$ $y^2 - x^2 = 1$
(2 points)



- 22) Sketch the graph of the polar equation $r = -4\cos(\theta)$, where r and θ are polar coordinates. You may use either the Cartesian or polar graph paper below; box in the one you use. Use arrows to indicate orientation. (8 points)

