Math 141 Name: ________________________

FINAL

(CHAPTERS 7-9)
MATH 141 – SPRING 2018 – KUNIYUKI
250 POINTS TOTAL

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed. A scientific calculator is allowed.

To maximize chances for partial credit, please be neat and indicate any elementary row operations (EROs) you use! Clarity is important. I might not grade "messes."

1) Find the intersection point(s) of the graphs of $y = 3x^2 + 2x - 2$ and $y = 2x^2 + 7x - 6$ in the usual xy-plane by solving a system, as in class. Do **not** rely on graphing, "trial-and-error," guessing, or point-plotting as a basis for your method. Show all work! Write the solution set with all solutions as ordered pairs of the form (x, y). (14 points)

2) Find the intersection points of the graphs of $2x - y = 0$ and $12x^2 - y^2 = 16$ in the usual *xy*-plane by solving a system, as in class. Follow the same instructions as for Problem 1). There are two solutions; checking may help. (14 points)

3) Write the PFD (Partial Fraction Decomposition) for $\frac{7x+17}{x^2+6x+9}$.

You must find the unknowns in the PFD Form. Show all work, as in class!
(17 points)

4) Write the PFD (Partial Fraction Decomposition) for $\frac{9x^2-17x+10}{(x-3)(x^2+1)}$.

You <u>must</u> find the unknowns in the PFD Form. Show all work, as in class! (17 points)

5) Write the PFD (Partial Fraction Decomposition) Form for

$\frac{1}{x^2(x-5)(x^2+9)^2}$. Do <u>not</u> find the unknowns (A, B, etc.). (9 points)

6) Solve the system below using matrices and Gaussian Elimination with Back-Substitution (or Gauss-Jordan Elimination, if you prefer). Write your solution as an ordered triple of the form (x, y, z) in a solution set.

Clearly indicate the elementary row operations (EROs) you are applying. Your final matrix must be in row-echelon form. (30 points)

$$\begin{cases} -3x + 8y + z = -13 \\ x - 2y + 3z = -5 \\ 5x - 11y + 8z = -5 \end{cases}$$

<u>YOU MAY CONTINUE ON THE BACK OF THE TEST.</u>

7) Let $A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. (4 points)

a) Is A in row-echelon form? Box in one: Yes No

b) Is A in reduced row-echelon (RRE) form? Box in one: Yes No

8) Let $B = \begin{bmatrix} 1 & 0 & 5 & 4 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. (4 points)

a) Is B in row-echelon form? Box in one: Yes No

b) Is B in reduced row-echelon (RRE) form? Box in one: Yes No

9) Find the matrix product AB if: $A = \begin{bmatrix} 5 & -1 & 1 \\ -2 & 2 & 0 \\ 3 & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & 3 \\ 4 & -1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$.

(18 points)

10) A and B are matrices consisting of real numbers. A has size 7×3, and B has size $c \times 5$. (6 points total; 3 points each)

a) What number must c be in order for the matrix product AB to be defined?

b) If c is the correct answer to part a), what size will the matrix product AB be?

11) Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 3 & -1 \\ 0 & 2 & 5 \end{bmatrix}$. (25 points total)

a) Find $\det(A)$ using Sarrus's Rule, the method using diagonals. (10 points)

b) Find $\det(A)$ using the Expansion by Cofactors Method. Show all work, as in class. (15 points)

12) Simplify completely: $\dfrac{(3n+1)!}{(3n-1)!}$ $\left(n \in \mathbb{Z}^{+}\right)$. (4 points)

13) Evaluate: $\sum_{i=0}^{3} i!$. (7 points)

14) Consider the sequence: $1, -3, 5, -7, 9, -11, \ldots$.
Write a nonrecursive expression (formula) for the apparent general n^{th} term, a_n, for this sequence, as in class. Let a_1 be the initial term; that is, assume that n begins with 1. (5 points)

15) Follow the same instructions as for Problem 14), but consider the sequence: $\frac{1}{2}, \frac{4}{3}, \frac{9}{4}, \frac{16}{5}, \frac{25}{6}, \ldots$. Write a nonrecursive expression for the apparent general n^{th} term, a_n. (5 points)

16) Consider the sequence defined recursively as follows. a_1 is considered to be the first term. (7 points total)

$$\begin{cases} a_1 = 5 \\ a_{k+1} = 4a_k \quad \left(\forall k \in \mathbb{Z}^+\right) \end{cases}$$

a) Write the first four terms of the sequence. (5 points)

b) The sequence is …. Box in one: (2 points)

Arithmetic Geometric Neither

17) Consider the arithmetic sequence: 3, 10, 17, 24, 31, ... (10 points total)

a) Write a nonrecursive expression (formula) for the general n^{th} term, a_n, for this sequence. Let a_1 be the initial term; that is, assume that n begins with 1.

b) Find a_{50}.

18) Consider the geometric sequence: $10, \ -5, \ \frac{5}{2}, \ -\frac{5}{4}, \ \frac{5}{8}, \ \ldots.$ (16 points total)

a) Write a nonrecursive expression (formula) for the general n^{th} term, a_n, for this sequence. Let a_1 be the initial term; that is, assume that n begins with 1.

b) Find the sum of the corresponding geometric series, $\sum_{n=1}^{\infty} a_n$, which is:

$$10 - 5 + \frac{5}{2} - \frac{5}{4} + \frac{5}{8} - \ldots$$

c) Is the series in b) convergent or divergent? Box in one:

Convergent Divergent

19) For what real values of x does the series $\sum_{n=1}^{\infty} (7x)^n$ converge? Write your answer using interval form (the form using parentheses and/or brackets).
Note: The interval is called the "interval of convergence" for the series.
Hint: The series is geometric. (Let's say this is true even for $x = 0$.) (4 points)

20) Does the geometric series $1 - 1 + 1 - 1 + 1 - 1\ldots$ converge or diverge?
Box in one: It converges. It diverges.
(3 points)

21) Prove using mathematical induction: $1+2+3+...+n=\frac{n(n+1)}{2}$ $(\forall n\in\mathbb{Z}^{+})$.
(14 points)

22) Use the Binomial Theorem to expand and simplify: $(x+h)^5$. (12 points)

23) Let $f(x) = x^5$. Use your answer from 22) to evaluate and simplify the following difference quotient completely: $\frac{f(x+h) - f(x)}{h}$ $(h \neq 0)$. (5 points)