Math 141 Name: ________________________

FINAL

(CHAPTERS 7-10)

MATH 141 – SPRING 2023 – KUNIYUKI

250 POINTS TOTAL

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed. A scientific calculator is allowed.

To maximize chances for partial credit, please be neat and indicate any elementary row operations (EROs) you use! Clarity is important. I might not grade "messes."

1) Find the intersection point(s) of the graphs of $2x - y = 5$ and $x^2 + y^2 = 5$ in the usual xy-plane by solving a system, as in class. Do **not** rely on graphing, "trial-and-error," guessing, or point-plotting as a basis for your method. Show all work! Write the solution set; a solution must be written as an ordered pair of the form (x, y). (14 points)

2) Write the PFD (Partial Fraction Decomposition) Form for $\frac{1}{x^3(x+2)^2(x^2+4)}$. Do <u>not</u> find the unknowns (A, B, etc.). (8 points)

3) Write the PFD (Partial Fraction Decomposition) for $\frac{-7t-24}{t^2-t-6}$. You must find the unknowns in the PFD Form. Show all work, as in class! (17 points)

4) Write the PFD (Partial Fraction Decomposition) for $\dfrac{5x^3 - 8x^2 + 28x - 35}{x^4 + 7x^2}$.

You must find the unknowns in the PFD Form. Show all work, as in class! (25 points)

5) Let $A = \begin{bmatrix} 1 & 5 & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. (4 points)

a) Is A in row-echelon form? Box in one: Yes No

b) Is A in reduced row-echelon (RRE) form? Box in one: Yes No

6) Solve the system below using matrices and Gaussian Elimination with Back-Substitution (or Gauss-Jordan Elimination, if you prefer).

Write your solution as an ordered triple of the form (x, y, z) in a solution set.

Clearly indicate the elementary row operations (EROs) you are applying. **Your final matrix must be in row-echelon form.** (25 points)

$$\begin{cases} x-3y+4z=-1 \\ 2x-y+z=9 \\ 3x-8y+10z=1 \end{cases}$$

<u>YOU MAY CONTINUE ON THE BACK OF THIS TEST.</u>

7) Find the matrix A^2, which is AA, if $A = \begin{bmatrix} 2 & 0 & 3 \\ 1 & 4 & -1 \\ 3 & 2 & 0 \end{bmatrix}$. (12 points)

8) A and B are matrices consisting of real numbers. A has size 4×5, and B has size 3×4. For each part below, if the matrix expression is undefined (due to the given sizes), just write "Undefined." If the matrix expression is defined, write the size that the resulting matrix would have to be. (6 points total; 3 points each)

a) AB

b) BA

9) Let $A = \begin{bmatrix} 4 & 2 & 0 \\ -1 & 3 & 5 \\ -3 & 1 & 5 \end{bmatrix}$. (25 points total)

a) Find $\det(A)$ using Sarrus's Rule, the method using diagonals. (10 points)

b) Find $\det(A)$ using the Expansion by Cofactors Method.
Show all work, as in class. (15 points)

10) Consider the sequence: $-\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}, \frac{1}{6}, -\frac{1}{7}, \ldots.$

Write a nonrecursive expression (formula) for the apparent general n^{th} term, a_n, for this sequence, as in class. Let a_1 be the initial term; that is, assume that n begins with 1. (5 points)

11) Simplify completely: $\frac{(2n+3)!}{(2n)!}$ $\left(n \in \mathbb{Z}^+\right)$. You may leave your answer in factored form. (4 points)

12) Evaluate: $\sum_{k=1}^{3}(-3)^k$. (7 points)

13) Consider the arithmetic sequence: $4, -2, -8, -14, -20, \ldots.$ (10 points)

a) Write a nonrecursive expression (formula) for the general n^{th} term, a_n, for this sequence. Let a_1 be the initial term; that is, assume that n begins with 1.

b) Find a_{75}.

14) Consider the geometric sequence: $\frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \frac{2}{81}, \frac{2}{243}, \ldots$ (16 points)

a) Write a nonrecursive expression (formula) for the general n^{th} term, a_n, for this sequence, as in class. Let a_1 be the initial term; that is, assume that n begins with 1.

b) Find the sum of the corresponding geometric series, $\sum_{n=1}^{\infty} a_n$, where:

$$\sum_{n=1}^{\infty} a_n = \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \frac{2}{81} + \frac{2}{243} + \ldots$$

c) Is the series in b) convergent or divergent? Box in one:

Convergent Divergent

15) Does the infinite geometric series $3+3+3+3+3+3\ldots$ converge or diverge?
Box in one: It converges. It diverges. (3 points)

16) Consider the sequence defined recursively as follows.
a_1 is considered to be the first term. (6 points total)

$$\begin{cases} a_1 = 8 \\ a_{k+1} = a_k + 4 \quad \left(\forall k \in \mathbb{Z}^+\right) \end{cases}$$

a) Write the first four terms of the sequence. (4 points)

b) The sequence is …. Box in one: (2 points)

Arithmetic Geometric Neither

17) Prove using mathematical induction: $1+2+3+...+n=\frac{n(n+1)}{2}$ $\left(\forall n\in\mathbb{Z}^{+}\right)$.

(14 points)

18) Use the Binomial Theorem to expand and simplify: $(x+h)^4$. (10 points)

19) Let $f(x) = x^4$. Use your answer from 18) to evaluate and simplify the following difference quotient completely: $\frac{f(x+h) - f(x)}{h}$ $(h \neq 0)$. (5 points)

20) An ellipse has equation $9x^2 + 16y^2 - 18x + 64y - 71 = 0$ in the usual xy-plane. (26 points total)

a) Find the standard form of the equation of this ellipse. (11 points)

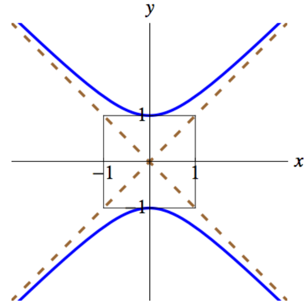
b) The center of this ellipse is at what point? (2 points)

c) The vertices of this ellipse are at what points? (4 points)

d) The foci of this ellipse are at what points? (6 points)

e) What is the eccentricity of this ellipse? (3 points)

21) The graph below is the graph of (box in one:) $x^2 - y^2 = 1$ $\quad$ $y^2 - x^2 = 1$ (2 points)



22) Sketch the graph of the polar equation $r = 4\sin(\theta)$, where r and θ are polar coordinates. You may use either the Cartesian or polar graph paper below; box in the one you use. Use arrows to indicate orientation. (6 points)