Math 141 Name: ________________________

FINAL

(CHAPTERS 7-10)
MATH 141 – SPRING 2024 – KUNIYUKI
250 POINTS TOTAL

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed. A scientific calculator is allowed.

To maximize chances for partial credit, please be neat and indicate any elementary row operations (EROs) you use! Clarity is important. I might not grade "messes."

1) Find the intersection point(s) of the graphs of $3x^2 - y = 0$ and $5x + y - 2 = 0$ in the usual xy-plane by solving a system, as in class. Do **not** rely on graphing, "trial-and-error," guessing, or point-plotting as a basis for your method. Show all work! Write the solution set with all solutions as ordered pairs of the form (x, y). (14 points)

2) Write the PFD (Partial Fraction Decomposition) for $\frac{-7t-24}{t^2-t-6}$. You must find the unknowns in the PFD Form. Show all work, as in class! (17 points)

3) Write the PFD (Partial Fraction Decomposition) for $\dfrac{5x^3 - 8x^2 + 28x - 35}{x^4 + 7x^2}$.

You <u>must</u> find the unknowns in the PFD Form. Show all work, as in class! (25 points)

4) Write the PFD (Partial Fraction Decomposition) Form for

$\dfrac{1}{x^3(x-6)(x^2+1)}$. Do <u>not</u> find the unknowns (A, B, etc.). (7 points)

5) Solve the system below using matrices and Gaussian Elimination with Back-Substitution (or Gauss-Jordan Elimination, if you prefer).

Write your solution as an ordered triple of the form (x, y, z) in a solution set.

Clearly indicate the elementary row operations (EROs) you are applying. **Your final matrix must be in row-echelon form.** (25 points)

$$\begin{cases} 4x + 6y + 15z = 40 \\ \quad - 3y + \ 9z = 21 \\ x + 2y + \ 2z = \ 6 \end{cases}$$

<u>**YOU MAY CONTINUE ON THE BACK OF THIS TEST.**</u>

6) Let $A = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. (4 points)

a) Is A in row-echelon form? Box in one: Yes No

b) Is A in reduced row-echelon (RRE) form? Box in one: Yes No

7) Find the matrix A^2 if $A = \begin{bmatrix} 1 & -4 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Hint: $A^2 = AA$. (12 points)

8) A and B are matrices consisting of real numbers. A has size 7×3, and B has size $c \times 5$. (6 points total; 3 points each)

a) What number must c be in order for the matrix product AB to be defined?

b) If c is the correct answer to part a), what size will the matrix product AB be?

9) Evaluate and simplify the determinant: $\begin{vmatrix} a & b \\ ac & bc \end{vmatrix}$, $\left(a, b, c \in \mathbb{R}\right)$. (4 points)

10) Let $A = \begin{bmatrix} 3 & 5 & 0 \\ -2 & -1 & 3 \\ 2 & 6 & 1 \end{bmatrix}$. Show all work, as in class. (23 points total)

a) Find $\det(A)$ using Sarrus's Rule, the method using diagonals. (10 points)

b) Find $\det(A)$ using the Expansion by Cofactors Method. (13 points)

11) Simplify completely: $\dfrac{(3n+1)!}{(3n-1)!}$ $\left(n \in \mathbb{Z}^+\right)$. You may leave your answer in factored form. (5 points)

12) Evaluate: $\sum_{k=2}^{4}\left(2^k - 1\right)$. (7 points)

13) Consider the sequence: $-\frac{1}{2}, \frac{1}{4}, -\frac{1}{6}, \frac{1}{8}, -\frac{1}{10}, ...$ Write a nonrecursive expression (formula) for the apparent general n^{th} term, a_n, for this sequence, as in class. Let a_1 be the initial term; that is, assume that n begins with 1. (5 points)

14) Consider the arithmetic sequence: 4, 9, 14, 19, 24, ... Write a nonrecursive expression (formula) for the general n^{th} term, a_n, for this sequence, as in class. Let a_1 be the initial term; that is, assume that n begins with 1. (6 points)

15) Consider the geometric sequence: $\frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \frac{2}{81}, \frac{2}{243}, ...$ (16 points)

a) Write a nonrecursive expression (formula) for the general n^{th} term, a_n, for this sequence, as in class. Let a_1 be the initial term; that is, assume that n begins with 1.

b) Find the sum of the corresponding geometric series, $\sum_{n=1}^{\infty} a_n$, where:

$$\sum_{n=1}^{\infty} a_n = \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \frac{2}{81} + \frac{2}{243} + ...$$

c) Is the series in b) convergent or divergent? Box in one:

Convergent　　　　　　　　Divergent

16) Consider the sequence recursively defined by: $\begin{cases} a_1 = 3 \\ a_2 = 10 \\ a_{k+2} = a_{k+1}a_k \quad (\forall k \in \mathbb{Z}^+) \end{cases}$

Find a_3, a_4, and a_5. (6 points)

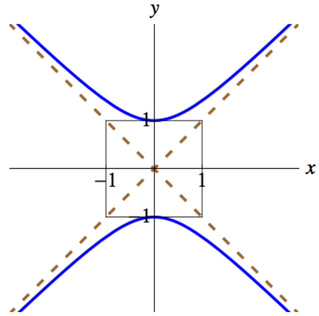
17) Prove using mathematical induction: $1+2+3+...+n = \frac{n(n+1)}{2}$ $(\forall n \in \mathbb{Z}^+)$.

(14 points)

18) Use the Binomial Theorem to expand and simplify, as in class: $(x+h)^4$.
(10 points)

19) Let $f(x)=x^4$. Use your answer from 18) to evaluate and simplify the following difference quotient completely: $\dfrac{f(x+h)-f(x)}{h}$ $(h \neq 0)$
(5 points)

20) The graph below is the graph of (box in one:) $x^2 - y^2 = 1$ $y^2 - x^2 = 1$
(3 points)



21) An ellipse has equation $9x^2 + 16y^2 - 18x + 64y - 71 = 0$ in the usual xy-plane. (28 points total)

a) Find the standard form of the equation of this ellipse.
Show all work, as in class. (12 points)

b) The center of this ellipse is at what point? (2 points)

c) The vertices of this ellipse are at what points? (4 points)

d) The foci of this ellipse are at what points? (7 points)

e) What is the eccentricity of this ellipse? (3 points)

22) Sketch the graph of the polar equation $r = -3\sin(\theta)$, where r and θ are polar coordinates. You may use either the Cartesian or polar graph paper below; box in the one you use. Use arrows to indicate orientation. (8 points)