CHAPTER 1:

Functions

(A) means "refer to Part A," (B) means "refer to Part B," etc.
(Calculator) means "use a calculator." Otherwise, do not use a calculator.

SECTION 1.1: FUNCTIONS

#1-2. **Finding function rules.** Find a function rule of the form $f(x) =$ (expression in x) that is consistent with the input-output machines. (D)

1)

$5 \to \boxed{f} \to -5$ $\quad -\pi \to \boxed{f} \to \pi$ $\quad -14.7 \to \boxed{f} \to 14.7$

2)

$6 \to \boxed{f} \to 6$ $\quad -\sqrt{3} \to \boxed{f} \to \sqrt{3}$ $\quad -\frac{2}{3} \to \boxed{f} \to \frac{2}{3}$

#3-8. **Evaluating functions using tables.** Fill out the tables. Give exact values unless otherwise specified. (D, F)

3) Let $f(x) = x + 4$.
(*h* is a constant or a variable, not a function)

Input x	Output $f(x)$
-1	
0	
1	
2	
$\sqrt{5}$	
π	
$10/3$	
4.7	
c	
$a+h$	

4) Let $f(x) = 2x$.
(*h* is a constant or a variable, not a function)

Input x	Output $f(x)$
-1	
0	
1	
2	
$\sqrt{5}$	
π	
$10/3$	
4.7	
c	
$a+h$	

5) Let $g(t) = t^2 - 9t + 5$.

Input t	Output $g(t)$
-2	
-1	
0	
1	
2	
c	

6) Let $h(r) = 9 - r^4$.

Input r	Output $h(r)$
-2	
-1	
0	
1	
2	
c	

7) Let $V(r) = \frac{4}{3}\pi r^3$.

Input r	Output $V(r)$
1	
2	
$5/2$	
c	

(If $r > 0$, how can $V(r)$ be interpreted geometrically?)

8) Let $f(x) = 7$.

Input x	Output $f(x)$
1	
2	
$5/2$	
c	

#9-17. **Finding domains.** Write the indicated domain in … (G, H)

- Task a): … set-builder form (write $\mathbb{R}$ when appropriate)
- Task b): … graphical form
- Task c): … interval form

9) $\text{Dom}(f)$, where $f(x) = 8x^6 - 3x^2 + 2$

10) $\text{Dom}(f)$, where $f(x) = \dfrac{1}{x-4}$

11) $\text{Dom}(g)$, where $g(t) = \dfrac{t^2 + t}{t + \sqrt{3}}$

12) $\text{Dom}(g)$, where $g(t) = \dfrac{7\sqrt{t}}{2t-1}$

13) $\text{Dom}(h)$, where $h(r) = \sqrt{r+2}$

14) $\text{Dom}(h)$, where $h(r) = \sqrt[3]{r+2}$

15) $\text{Dom}(h)$, where $h(r) = (r+2)^{-1/2}$

16) $\text{Dom}(f)$, where $f(x) = \dfrac{\sqrt[4]{2-5x}}{\sqrt[3]{x+1}}$

17) $\text{Dom}(g)$, where $g(t) = \dfrac{t^4 - 5}{3t^2 - 5t - 2}$

#18-20. **Evaluating functions.** (I)

18) Let $f(x) = \sqrt{x}$.

Evaluate: a) $f(4)$, b) $f(11)$, c) $f(a)$, d) $f(x+h)$, e) $f(x^2 + 2x + 1)$.

19) Let $g(t) = 2t^2 - 3t - 7$.

Evaluate: a) $g(3)$, b) $g(x)$, c) $g(t^2)$, d) $g(t+h)$.

20) Let $h(x) = \dfrac{3}{2x+1}$.

Evaluate: a) $h(5)$, b) $h(\pi)$, c) $h(x+h)$

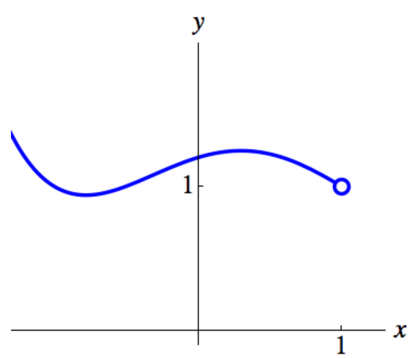
21) For a function f and some real value a, $f(a) = a^2$ and $f(a) = 6a - 9$. Find a. (C, D)

SECTION 1.2: GRAPHS OF FUNCTIONS

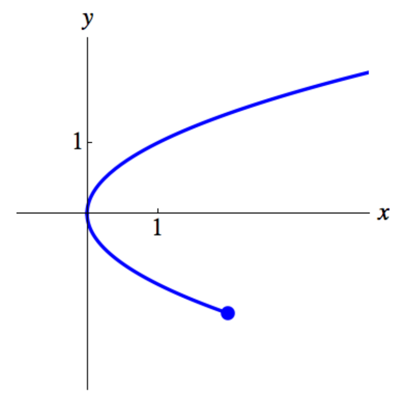
1) Let $h(r) = 9 - r^4$. Graph $p = h(r)$ in the *rp*-plane by using Point-Plotting and your table from Section 1.1, Exercise 6. (B, C, I)

2) For each graph below, write "Yes" if the graph describes *y* as a function of *x*, and "No" if it does not. (D)

a)

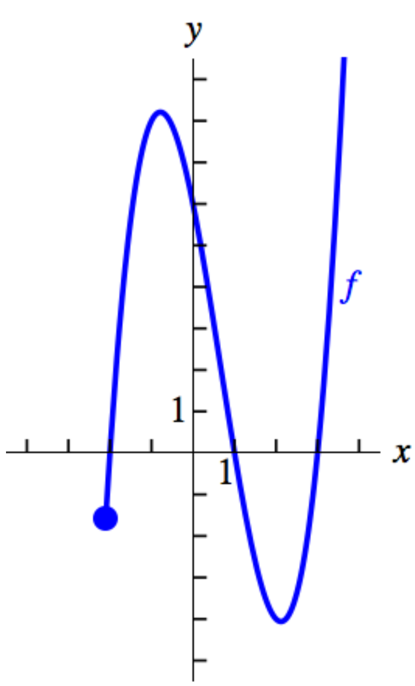


b)



3) Let $h(r) = 9 - r^4$, as in Exercise 1. Use your graph from Exercise 1 to estimate the domain and the range of *h*. Write them in interval form. (F)

4) The graph of *f*, or the graph of $y = f(x)$, is below. (F, G)



a) Estimate $f(0)$.

b) Estimate $f(-1)$.

c) Estimate the real zeros of f.

5) For each of a)-d) below, find the *x*- and *y*-intercepts of the graph of $y = f(x)$. If there are none, write "NONE." (G)

a) $f(x) = \dfrac{x-4}{x+2}$

b) $f(x) = \dfrac{3}{x^2+1}$

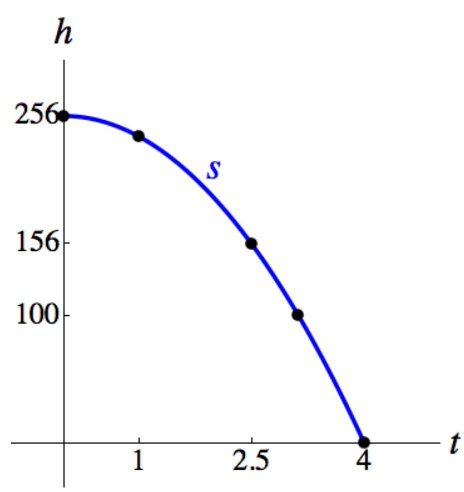
c) $f(x) = \sqrt{9-3x-4x^2}$

<u>Hint</u>: A positive integer is divisible by 3 if and only if the sum of its digits (including repetitions) is divisible by 3. A similar trick holds for divisibility by 9.

d) $f(x) = \dfrac{5x^2+19x-4}{x+4}$

6) Let $h(r) = 9 - r^4$, as in Exercise 1. Use your graph from Exercise 1 to determine the intervals of increase and intervals of decrease for *h*. (H)

7) (Falling coin problem). In Example 13, we discussed a position function *s* for a falling coin you dropped from the top of a building. *t* is the time elapsed (in seconds) since you dropped the coin. *h* is the height (in feet) of the coin. Use the given graph of $h = s(t)$ below to estimate answers to the following questions. (D, F, I)



a) Why is *s* a function?

b) What is $\text{Dom}(s)$, the domain of *s*?

c) What is the range of s?

d) How tall is the building? Assume that you were lying on your stomach (with a horizontally outstretched arm) at the top of the building when you dropped the coin.

e) From the time it was dropped, how long did it take for the coin to hit the ground?

f) Why is the graph concave down (curving downward)?

g) The point (2.5, 156) lies on the graph. Interpret this.

• We are now informed that $s(t) = -16t^2 + 256$.

h) How high was the coin one second after it was dropped? Check the graph to see if your answer makes sense.

i) From the time it was dropped, how long did it take for the coin to be 100 feet above the ground? Give an exact answer and an approximate answer rounded off to three significant digits. Check the graph to see if your approximate answer makes sense. (Calculator)

j) Verify your answer to e) by solving an equation.

SECTION 1.3: BASIC GRAPHS and SYMMETRY

KNOW ALL OF THE BASIC FUNCTIONS AND GRAPHS IN THIS SECTION. ALSO KNOW THEIR DOMAINS, RANGES, AND SYMMETRIES.

1) Let $f(x) = x^{4/5}$. (E, F, G)

   a) Is f even, odd, or neither? Give a proof.

   b) Fill in the blank: The graph of $y = f(x)$ is symmetric about ____________ .

2) Let $g(t) = t^5 + t^3 + t$. (E, F, G)

   a) Is g even, odd, or neither? Give a proof.

   b) Fill in the blank: The graph of $y = g(t)$ is symmetric about ____________ .

3) (Falling coin problem.) Refer to Section 1.2, Exercise 7. Is s an even function? Why or why not? (E, F, G)

4) Why is it rare for the graph of $y = f(x)$ to be symmetric about the x-axis?

SECTION 1.4: TRANSFORMATIONS

1) Graph the following in the *xy*-plane. Identify any intercepts. (B-D)

   a) $y=(x-3)^2$

   b) $y=(x+2)^3$

   c) $y=x^4-1$

   d) $y=\dfrac{1}{x-4}$

   e) $y=-x^{2/3}$

   f) $y=\sqrt{16-x^2}+2$

2) Graph the following in the *xy*-plane. (B-E)

   a) $y=\dfrac{1}{x-1}+3$

   b) $y=2-\dfrac{1}{(x+3)^2}$

   c) $y=-\sqrt{x}-4$

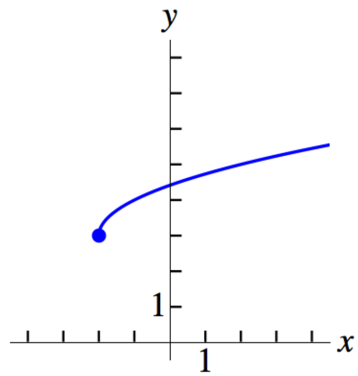
   d) $y=\sqrt[3]{x-2}+1$

   e) $y=-2|x|$

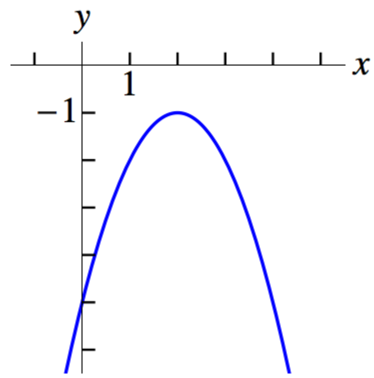
3) (Falling coin problem.) Refer to Section 1.2, Exercise 7. Your coin's height follows the rule $s(t)=-16t^2+256$. If your friend drops a coin off the top of the same building one second after you do, what should be the rule for $S(t)$, the height of your friend's coin? What should $\text{Dom}(S)$ be? Sketch the graph of $h=S(t)$.

4) The graphs below are obtained by taking basic graphs from Section 1.3 and applying rigid transformations. Find an equation for each graph. (B-E)

a)



b)



5) (B-E). If the point $(-4, 2)$ lies on the graph of $y = f(x)$, what point must then lie on the graph of …

a) … $y = f(x+2) - 5$?

b) … $y = f(x-3) + 6$?

c) … $y = -f(x)$?

d) … $y = f(-x)$?

e) … $y = 3f(x)$?

6) A function f has 3 and 9 as its only zeros. Using only this information, what do we know about the zeros of the following functions? If we know nothing, write "NOTHING." Hint: Consider x-intercepts. (B, C, D)

a) g, where $g(x) = f(x-2)$

b) h, where $h(x) = f(x) + 1$

c) p, where $p(x) = f(3x)$

d) q, where $q(x) = 2f(x)$

e) r, where $r(x) = -f(x)$

f) s, where $s(x) = f(-x)$

SECTION 1.5: PIECEWISE-DEFINED FUNCTIONS; LIMITS AND CONTINUITY IN CALCULUS

1) (B-D). Let the function f be piecewise-defined by:

$$f(x) = \begin{cases} |x+2|, & -4 < x \le 1 \\ \sqrt{x-1}, & 1 < x < 5 \end{cases}$$

a) Evaluate $f(-3)$.

b) Evaluate $f(1)$.

c) Evaluate $f(3)$.

d) Evaluate $f(5)$.

e) Graph $y = f(x)$.

f) What is the domain of f?

g) What is the range of f?

2) (B-D). Let the function g be piecewise-defined by:

$$g(x) = \begin{cases} 2, & x < -1 \\ 3x+1, & -1 \le x < 0 \\ x^2 - 2, & 1 \le x \le 3 \end{cases}$$

a) Evaluate $g(-3)$.

b) Evaluate $g(-1)$.

c) Evaluate $g(0)$

d) Evaluate $g(3)$.

e) Graph $y = g(x)$.

f) What is the domain of g?

g) What is the range of g?

3) Evaluate the following. (E)

a) $[\![2.3]\!]$, also written as $\lfloor 2.3 \rfloor$

b) $[\![5.7]\!]$, also written as $\lfloor 5.7 \rfloor$

c) $[\![-4]\!]$, also written as $\lfloor -4 \rfloor$

d) $[\![-6.8]\!]$, also written as $\lfloor -6.8 \rfloor$

SECTION 1.6: COMBINING FUNCTIONS

1) Let $f(x)=\sqrt{x}$ and $g(x)=\sqrt{x}-4$. (B, C, D)

   a) Find $(f+g)(x)$ and $\text{Dom}(f+g)$; use interval form.

   b) Find $(f-g)(x)$ and $\text{Dom}(f-g)$; use interval form.

   c) Find $(fg)(x)$ and $\text{Dom}(fg)$; use interval form.

   d) Find $\left(\dfrac{f}{g}\right)(x)$ and $\text{Dom}\left(\dfrac{f}{g}\right)$; use interval form.

   e) Let $h=2f+3g$, a linear combination of f and g. Find $h(x)$ and $\text{Dom}(h)$; use interval form.

   f) Find $(f\circ g)(x)$.

   ADDITIONAL PROBLEM: Find $\text{Dom}(f\circ g)$; use interval form.

   g) Evaluate $(f\circ g)(64)$.

2) x is tripled, and the result is squared to obtain the final result. If x is squared first, and if the result is tripled, do we obtain the same final result? What does this tell us about $f\circ g$ and $g\circ f$ for functions f and g? (D)

3) Let $f(x)=x^2$ and $g(x)=\dfrac{1}{x-9}$. (B, D)

   a) Find $(f\circ g)(x)$.

   ADDITIONAL PROBLEM: Find $\text{Dom}(f\circ g)$; use interval form.

   b) Find $(g\circ f)(x)$.

   ADDITIONAL PROBLEM: Find $\text{Dom}(g\circ f)$; use interval form.

   c) In this Exercise, is it true that $f\circ g=g\circ f$?

4) For each of a)-c) below, find $g(x)$ and $f(u)$ such that $(f\circ g)(x)$ is as indicated. We want to "decompose" $f\circ g$. Neither f nor g may be an identity function. (E)

   a) $(f\circ g)(x)=\sqrt[3]{x^2-7}$

   b) $(f\circ g)(x)=(4x+3)^8$

   c) $(f\circ g)(x)=\dfrac{5}{2x-9}$

SECTION 1.7: SYMMETRY REVISITED

1) For each function f below, state whether the function is even, odd, or neither. Also state whether the graph of $y = f(x)$ is symmetric about the y-axis, the origin, or neither. (B, C)

   a) f, where $f(x) = 8x^7 - 6x^3 + 3x$

   b) f, where $f(x) = 5x^6 + 3x^4 - 1$

   c) f, where $f(x) = x^4 + 7x^3 + x^2$

   d) f, where $f(x) = -3x^5 + x^3 - 7x + 2$

2) For each function below, state whether the function is even, odd, or neither. If the function is even or odd, prove it. If the function is neither, show why it is neither even nor odd. (B, C)

   a) f, where $f(x) = 3x^4 + 2x^2 + 7 - \frac{1}{x^2}$

   b) f, where $f(t) = \sqrt{t}$

   c) g, where $g(x) = 4x^3 - x + \frac{2}{x^3} + \sqrt[3]{x}$

   d) h, where $h(r) = r^3 + 1$

SECTION 1.8: $x = f(y)$

1) Graph the following. (B, C, G, H)

   a) $x = y^2 + 1$

   b) $x = y^2 - 3$

   c) $x = 4 - y^2$

   d) $x = (y-1)^2$

   e) $x = \sqrt{y}$. Hint: This is equivalent to: $y = x^2$, $(x \geq 0)$.

SECTION 1.9: INVERSES OF ONE-TO-ONE FUNCTIONS

1) Let $f(x) = \sqrt[5]{x}$ on $\mathbb{R}$. Find f^{-1}. (B)

2) Refer to Example 2c. The table below gives the input-output pairs for f, where $f(x) = x^3$ on the domain $\{-2, 1, \pi\}$. (B)

f	
Input x	Output $f(x)$
-2	-8
1	1
π	π^3

a) Write the table for f^{-1}.

b) Observe that $f(-2) = -8$. Evaluate $f^{-1}(-8)$.

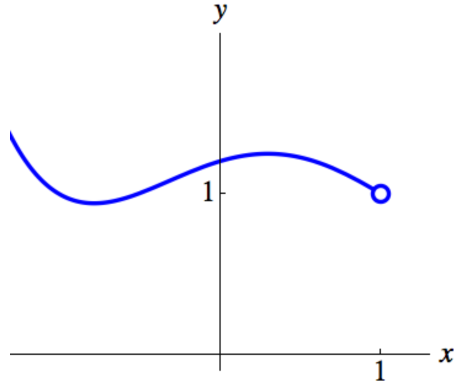
c) Find the range of f^{-1}.

d) Evaluate $f^{-1}(f(\pi))$.

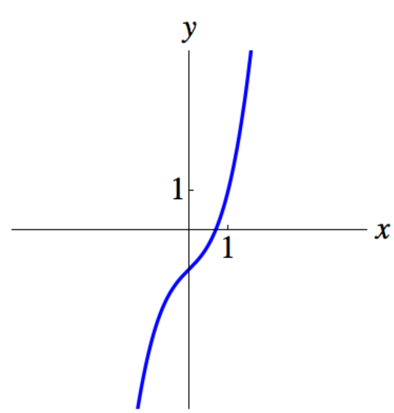
e) Evaluate $f(f^{-1}(-8))$

3) For each graph below, write "Yes" if it describes y as a one-to-one (and therefore invertible) function of x, and "No" otherwise. (C)

a)



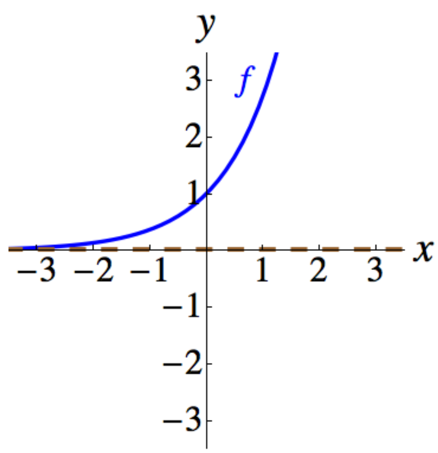
b)



4) Yes or No: If a function f is increasing on $\mathbb{R}$, then must f be one-to-one and invertible? (C)

5) If the point $(-4, 2)$ lies on the graph of $y = f(x)$, where f is a one-to-one function, what point must then lie on the graph of $y = f^{-1}(x)$? (D)

6) A one-to-one function f is graphed below. Graph its inverse. (D)



7) Let $f(x) = \left(\frac{3x+1}{8} \right)^5$ on $\mathbb{R}$. Find $f^{-1}(x)$. (E)

8) Let $g(x) = 2\left(\sqrt[3]{4x-5}\right)$ on $\mathbb{R}$. Find $g^{-1}(x)$ and $g^{-1}(6)$. (E)

9) (Temperature conversion). Let f be the one-to-one function that converts from Celsius to Fahrenheit. Refer to Example 10, which also discusses domains, ranges, and graphs. (E)

   a) Develop a linear rule for $f(x)$ such that $f(0) = 32$ and $f(100) = 212$.

   b) Find $f^{-1}(x)$ in three different ways:

      i. Develop a linear rule for $f^{-1}(x)$ such that $f^{-1}(32) = 0$ and $f^{-1}(212) = 100$.

      ii. Begin with your rule for $f(x)$ from a) and apply the Conceptual Approach used in Example 6.

      iii. Begin with your rule for $f(x)$ from a) and apply the Mechanical Approach used in Example 7.

SECTION 1.10: DIFFERENCE QUOTIENTS

1) Let $f(x) = x^4$. Find the average rate of change of f on $[-3, 2]$. (B-E)

2) The position function s for a particle is defined by: $s(t) = t^3 - 2t + 1$ on $[0, \infty)$, where position s is measured in meters and time elapsed t is measured in seconds. Find the average velocity of the particle between $t = 3$ seconds and $t = 7$ seconds. (Calculator) (B-E)

3) (Falling coin problem). Refer to Section 1.2, Exercise 7. Find the average velocity of the coin between the time it is dropped and the time it hits the ground. Why is it negative? (B-E)

4) (E). Let $f(x) = \frac{1}{x}$. Simplify the difference quotient completely:

$$\frac{f(x+h) - f(x)}{h} \quad (h \neq 0)$$

5) (E). Let $f(x) = \sqrt{x}$. Rationalize the numerator and simplify the difference quotient completely:

$$\frac{f(x+h) - f(x)}{h} \quad (h \neq 0)$$

6) (E). Let $g(t) = 5t^2 + 3t - 2$. Simplify the difference quotient completely:

$$\frac{g(t+h) - g(t)}{h} \quad (h \neq 0)$$

SECTION 1.11: LIMITS AND DERIVATIVES IN CALCULUS

1) **ADDITIONAL PROBLEMS.** Find the indicated derivatives by using the corresponding difference quotients from Section 1.10. (D)

   a) Let $f(x) = \frac{1}{x}$. Find $f'(x)$ by using Section 1.10, Exercise 4.

   b) Let $f(x) = \sqrt{x}$. Find $f'(x)$ by using Section 1.10, Exercise 5.

   c) Let $g(t) = 5t^2 + 3t - 2$. Find $g'(t)$ by using Section 1.10, Exercise 6.