CHAPTER 7:
Systems and Inequalities

(A) means "refer to Part A," (B) means "refer to Part B," etc.
(Calculator) means "use a calculator." Otherwise, do not use a calculator.

SECTIONS 7.1-7.3: SYSTEMS OF EQUATIONS

When solving a system, only give solutions in $\mathbb{R}^2$, the set of ordered pairs of real numbers. All such solutions correspond to intersection points of the graphs of the given equations. If there are no such solutions, write $\varnothing$, the empty set or null set.

Write solutions in a solution set as ordered pairs of the form (x, y). Unless otherwise specified, do **not** rely on graphing or "trial-and-error point-plotting."

1) Consider the system $\begin{cases} x + y = 5 \\ 5x - 3y = -23 \end{cases}$. (A-E)

   a) The graphs of the equations in the system are distinct lines in the *xy*-plane that are not parallel. How many solutions does this system have?

   b) Solve the system using the Substitution Method.

   c) Solve the system using the Addition / Elimination Method.

2) Consider the system $\begin{cases} x^2 + y^2 = 2 \\ y = x + 2 \end{cases}$. (A-D)

   a) Find the solution set of the system.

   b) Use the solution set from a) to graph the equations in the system in the usual *xy*-plane.

3) Consider the system $\begin{cases} x^2 + y^2 = \dfrac{3}{2} \\ y\sqrt{2} = x^2 \end{cases}$. (A-D)

   a) What are the graphs of the equations in the system in the usual *xy*-plane?

   b) How many solutions does the system have?

   c) Find the solution set of the system.

4) Consider the system $\begin{cases} x = y^2 \\ x = 4 - y^2 \end{cases}$. (A-D; Section 1.8)

a) What are the graphs of the equations in the system in the usual *xy*-plane?

b) How many solutions does the system have?

c) Find the solution set of the system.

5) Consider the system $\begin{cases} x^2 + y = 0 \\ y - x^2 = 1 \end{cases}$. (A-D, F)

a) Sketch graphs of the equations in the system in the usual *xy*-plane.

b) Based on your graphs in a), find the solution set of the system.

c) Verify the solution set by using the Substitution Method or the Addition / Elimination Method to solve the system.

6) Solve the following systems. (A-D, F)

a) $\begin{cases} y = 3x^2 - x \\ y = 2x^2 - 3x + 8 \end{cases}$

b) $\begin{cases} x^2 + 4y^2 = 2 \\ 3x - 2y = -4 \end{cases}$

c) $\begin{cases} x^2 + y^2 = 1 \\ x^2 - y^2 = 4 \end{cases}$

7) **ADDITIONAL PROBLEM.** Solve the system $\begin{cases} 0 = 0 \\ 0 = 1 \end{cases}$. (A-D, F)

SECTION 7.4: PARTIAL FRACTIONS

1) Write the PFD (Partial Fraction Decomposition) Form for the following.
Do not find the unknowns (A, B, etc.). (A-C)

a) $\dfrac{1}{(x+4)(x-3)(x^2+1)}$

b) $\dfrac{x+5}{x^3(x-1)^2(x^2+3)^2}$

c) $\dfrac{3t^2+2t-2}{t^2(2t+5)^3(2t^2+5)(t^2+t+1)}$

2) Write the PFD (Partial Fraction Decomposition) for the following. (A-G)

a) $\dfrac{3x-5}{x^2-5x+6}$

b) $\dfrac{2x^2-3x+19}{x^3+4x^2-7x-10}$. (Hint: Use the Rational Zero Test and Synthetic Division.)

c) $\dfrac{9x^2+14x+6}{2x^3+x^2}$

d) $\dfrac{x+1}{x^2-8x+16}$

e) $\dfrac{8x^2+7x+12}{(x+2)(x^2+1)}$

f) $\dfrac{5x^2-5x+12}{x^3-5x^2+3x-15}$. (Hint: Use Factoring by Grouping.)

g) $\dfrac{-5x^2-8x-3}{x^3+x^2+x}$

h) $\dfrac{5t^3-t^2+20t-8}{(t^2+4)^2}$

3) A student writes: $\dfrac{x^4}{(x+3)(x+5)} = \dfrac{A}{x+3} + \dfrac{B}{x+5}$. Is this appropriate? Why or why not?

CHAPTER 8:

Matrices and Determinants

(A) means "refer to Part A," (B) means "refer to Part B," etc.
Most of these exercises can be done without a calculator, though one may be permitted.

SECTION 8.1: MATRICES and SYSTEMS OF EQUATIONS

1) Give the size and the number of entries (or elements) for each matrix below. (A)

a) $\begin{bmatrix} 14 & 1/5 \\ -2 & \sqrt{3} \\ \pi & -4.7 \end{bmatrix}$

b) $\begin{bmatrix} 3 & 1 & 0 & 0 & 5 \\ -4 & 2/3 & 9 & 0 & 12 \\ 13 & 0 & -1/2 & -11 & 14 \\ 2 & 0 & 0 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$

2) Consider the system $\begin{cases} 3x - \ \ y = 18 \\ x + 2y = -1 \end{cases}$. (A-D)

a) Write the augmented matrix for the given system.
b) What size is the coefficient matrix?
c) What size is the right-hand side (RHS)?
d) Switch Row 1 and Row 2 of the augmented matrix. Write the new matrix.
e) Take the matrix from d) and add (-3) times Row 1 to Row 2. Write the new matrix.
f) Take the matrix from e) and divide Row 2 by (-7); that is, multiply Row 2 by $\left(-\frac{1}{7}\right)$. Write the new matrix, which will be in Row-Echelon Form (Part F).
g) Write the system corresponding to the matrix from f).
h) Solve the system from g) using Back-Substitution, and write the solution set.
i) Check your solution in the original system. (This is typically an optional step.)

In Exercises 3-12, use matrices and Gaussian Elimination with Back-Substitution.

3) Solve the system $\begin{cases} 4x+2y=-3 \\ x+\ \ y=-2 \end{cases}$. (A-D)

4) Solve the system $\begin{cases} 3x_1-9x_2=57 \\ -5x_1+4x_2=-18 \end{cases}$. (A-D)

5) **ADDITIONAL PROBLEM:** Solve the system $\begin{cases} -2a+3b=-\dfrac{16}{3} \\ \dfrac{1}{2}a-4b=\dfrac{17}{3} \end{cases}$. (A-D)

6) Solve the system $\begin{cases} x+3y=6 \\ -2x-6y=-9 \end{cases}$. (A-E)

7) Consider the system $\begin{cases} x+3y=6 \\ -2x-6y=-12 \end{cases}$. How many solutions does the system have? (A-E)

8) Solve the system $\begin{cases} y-2z=14 \\ x+z=-3 \\ 4x+6z=-22 \end{cases}$. Begin by rewriting the system. (A-D)

9) Solve the system $\begin{cases} 3x-10y+9z=50 \\ -2x+\ \ 6y-\ \ z=-27 \\ x-\ \ 2y-\ \ z=10 \end{cases}$. (A-D)

10) Solve the system $\begin{cases} a-4b-3c=-5 \\ a-4b-\ \ c=-2 \\ 2a-7b-4c=-7 \end{cases}$. (A-D)

11) Solve the system $\begin{cases} -2x_1+8x_2-10x_3=20 \\ 3x_1+5x_2+\ \ x_3=-5 \\ -4x_1-\ \ x_2+3x_3=-12 \end{cases}$. (A-D)

12) Solve the system $\begin{cases} x_1-\ \ 2x_2-\ \ x_3=3 \\ 5x_1-10x_2-5x_3=11 \\ 4x_1+\ \ 3x_2+2x_3=-18 \end{cases}$. (A-E)

13) Consider the augmented matrix $\left[\begin{array}{cccc|c} 1 & 0 & -7 & 0 & 1 \\ 0 & 1 & 3 & 0 & -2 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$. (F, G)

a) Is the matrix in Row-Echelon Form?
b) Is the matrix in Reduced Row-Echelon (RRE) Form?

14) Consider the augmented matrix $\left[\begin{array}{cccc|c} 1 & 4 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$. (F, G)

a) Is the matrix in Row-Echelon Form?
b) Is the matrix in Reduced Row-Echelon (RRE) Form?

15) Consider the augmented matrix $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 3 \\ 0 & 0 & 3 & 6 \end{array}\right]$. (F, G)

a) Is the matrix in Row-Echelon Form?
b) Is the matrix in Reduced Row-Echelon (RRE) Form?

16) Consider the augmented matrix $\left[\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$. (F, G)

a) Is the matrix in Row-Echelon Form?
b) Is the matrix in Reduced Row-Echelon (RRE) Form?

17) **ADDITIONAL PROBLEM:** Solve the systems below using Gauss-Jordan Elimination. (A-H)

a) $\begin{cases} 2x + 6y = -6 \\ 4x + 13y = -14 \end{cases}$; b) $\begin{cases} x - z = 3 \\ 2y = 16 \\ 3x + z = 13 \end{cases}$ (Rewrite first); c) $\begin{cases} 4x_1 + 4x_2 - 3x_3 = 7 \\ 5x_1 + 7x_2 - 13x_3 = -9 \\ x_1 + 2x_2 - 5x_3 = -6 \end{cases}$

SECTION 8.2: OPERATIONS WITH MATRICES

Assume that all entries (i.e., elements) of all matrices discussed here are real numbers.

1) Let $A = \begin{bmatrix} -1 & 3 \\ 2 & 4 \\ 7 & \pi \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 0 \\ 8 & -3 \\ -9 & \sqrt{5} \end{bmatrix}$. (C)
   a) Find $A + B$.
   b) Find $3A - 4B$.

2) Let $A = \begin{bmatrix} 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$. Find AB. (D)

3) Let $A = \begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$. (B, D, E)
   a) Find AB.
   b) Find BA.
   c) Yes or No: Is $AB = BA$ here?

4) Let $C = \begin{bmatrix} 2 & 0 & 7 \\ -1 & 4 & -3 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 6 & 4 \\ -2 & 3 & 1 \\ 2 & -1 & 0 \end{bmatrix}$. Find the indicated matrix products.
   If the matrix product is undefined, write "Undefined." (D, E)
   a) CD
   b) DC
   c) D^2, which is defined to be DD

5) Assume that A is an 8×10 matrix and B is a 10×7 matrix. (D, E)
   a) What is the size of the matrix AB?
   b) Let $C = AB$. Explain how to obtain the matrix element c_{56}.

6) If A is an $m \times n$ matrix and B is a $p \times q$ matrix, under what conditions are both AB and BA defined? (E)

7) Let $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$. This is called a <u>diagonal matrix</u>. (D, E)
   a) Find D^2.
   b) Based on a), conjecture (guess) what D^{10} is. (Calculator)

8) Assume that A is a 3×4 matrix, B is a 3×4 matrix, and C is a 4×7 matrix. Find the sizes of the indicated matrices. If the matrix expression is undefined, write "Undefined." (C-E)

   a) $A+4B$; b) $A-C$; c) AB; d) AC; e) $AC+BC$; f) $(A+B)C$

9) Write the identity matrix I_4. (F)

10) **ADDITIONAL PROBLEM:** In Section 8.1, Exercise 17c, you solved the system $\begin{cases} 4x_1 + 4x_2 - 3x_3 = 7 \\ 5x_1 + 7x_2 - 13x_3 = -9 \\ x_1 + 2x_2 - 5x_3 = -6 \end{cases}$. This system can be written in the form $AX = B$. (G)

   a) Identify A.
   b) Identify X (in the given system, before it is solved).
   c) Identify B.
   d) When you solved this system using Gauss-Jordan Elimination, what was the coefficient matrix of the final augmented matrix in Reduced Row-Echelon (RRE) Form? What was the right-hand side (RHS) of that matrix?

SECTION 8.3: THE INVERSE OF A SQUARE MATRIX (THESE ARE ALL ADDITIONAL PROBLEMS)

Assume that all entries (i.e., elements) of all matrices discussed here are real numbers.

1) If A is an invertible 2×2 matrix, what is AA^{-1}? (B)

2) Find the indicated inverse matrices using Gauss-Jordan Elimination. If the inverse matrix does not exist, write "A is noninvertible." (C)

   a) A^{-1}, where $A = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$. (Also check by finding AA^{-1}.)

   b) A^{-1}, where $A = \begin{bmatrix} 2 & 4 \\ 6 & 12 \end{bmatrix}$.

   c) A^{-1}, where $A = \begin{bmatrix} 0 & 4 & 8 \\ 0 & 0 & 3 \\ -5 & 0 & 0 \end{bmatrix}$

   d) A^{-1}, where $A = \begin{bmatrix} 2 & 13 & -5 \\ 1 & 5 & 2 \\ -1 & -2 & -8 \end{bmatrix}$

3) Use Exercise 2a to solve the system $\begin{cases} 3x_1 - x_2 = 15 \\ x_1 + 2x_2 = -2 \end{cases}$. (D)

4) Use Exercise 2d to solve the system $\begin{cases} 2x_1 + 13x_2 - 5x_3 = 7 \\ x_1 + 5x_2 + 2x_3 = -1 \\ -x_1 - 2x_2 - 8x_3 = 7 \end{cases}$. (D)

5) Let $A = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$, as in Exercise 2a. (E, F)

   a) Find $\det(A)$.

   b) Find A^{-1} using the shortcut from Part F. Compare with your answer to Exercise 2a.

6) Let $A = \begin{bmatrix} 2 & 4 \\ 6 & 12 \end{bmatrix}$, as in Exercise 2b. (E, F)

   a) Find $\det(A)$.

   b) What do we then know about A^{-1}? Compare with your answer to Exercise 2b.

7) Assume that A and B are invertible $n \times n$ matrices. Prove that $(AB)^{-1} = B^{-1}A^{-1}$. (B)

8) Verify that the shortcut formula for A^{-1} given in Part F does, in fact, give the inverse of a matrix A, where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $\det(A) \neq 0$. (E, F)

SECTION 8.4: THE DETERMINANT OF A SQUARE MATRIX

Assume that all entries (i.e., elements) of all matrices discussed here are real numbers, unless otherwise indicated.

1) Find the indicated determinants. (B)

   a) Let $A = [-4]$. Find $\det(A)$, or $|A|$.

   b) Let $B = \begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}$. Find $\det(B)$, or $|B|$.

   c) Let $C = \begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix}$. Find $\det(C)$, or $|C|$. Compare with b).

   d) Let $D = \begin{bmatrix} 30 & 20 \\ 5 & 4 \end{bmatrix}$. Find $\det(D)$, or $|D|$. Compare with b).

e) Let $E = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$. Find $\det(E)$, or $|E|$. Compare with b).

Note: $E = B^T$, the <u>transpose</u> of B. Rows become columns, and vice-versa.

f) Find $\begin{vmatrix} 2 & -4 \\ -3 & -7 \end{vmatrix}$.

g) Find $\begin{vmatrix} 4 & 5 \\ 0 & 0 \end{vmatrix}$.

h) Find $\begin{vmatrix} 4 & 5 \\ 40 & 50 \end{vmatrix}$.

i) Find $\begin{vmatrix} a & b \\ ca & cb \end{vmatrix}$. Compare with g) and h).

j) Find $\begin{vmatrix} 1 & e^x \\ x & e^{2x} \end{vmatrix}$. Here, the entries correspond to functions.

k) Find $\begin{vmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{vmatrix}$. Here, the entries correspond to functions.

2) Let $A = \begin{bmatrix} 2 & 4 & 3 \\ -1 & -3 & 3 \\ 5 & 1 & 2 \end{bmatrix}$. We will find $\det(A)$, or $|A|$, in three different ways. (B, C)

a) Use Sarrus's Rule, the shortcut for finding the determinant of a 3×3 matrix.
b) Expand by cofactors along the first row.
c) Expand by cofactors along the second column.

3) Let $B = \begin{bmatrix} 1 & 5 & -2 \\ 3 & 4 & 0 \\ -1 & -4 & 3 \end{bmatrix}$. We will find $\det(B)$, or $|B|$, in three different ways. (B, C)

a) Use Sarrus's Rule, the shortcut for finding the determinant of a 3×3 matrix.
b) Expand by cofactors along the second row.
c) Expand by cofactors along the third column.

4) Find $\begin{vmatrix} 1 & 2 & 3 \\ 10 & 20 & 30 \\ 4 & 5 & 6 \end{vmatrix}$. Based on this exercise and Exercise 1i, make a conjecture (guess) about determinants. (B, C)

5) Let $C = \begin{bmatrix} 10 & 400 & 500 \\ 0 & 20 & 600 \\ 0 & 0 & 30 \end{bmatrix}$. C is called an <u>upper triangular matrix</u>. Find $\det(C)$, or $|C|$. Based on this exercise, make a conjecture (guess) about determinants of upper triangular matrices such as C. What about lower triangular matrices? (B, C)

6) Find $\begin{vmatrix} -1 & 4 & 0 & -2 \\ 3 & 2 & 0 & 1 \\ -1 & 3 & 0 & -4 \\ 0 & -2 & 2 & 1 \end{vmatrix}$. (B, C)

7) Find $\begin{vmatrix} 13 & 42 & -\pi & \sqrt[3]{5} \\ 92 & 5 & -3.2 & e \\ e^2 & \sqrt{\pi} & 267 & 9876 \\ 0 & 0 & 0 & 0 \end{vmatrix}$. (C)

8) Solve the equation $\begin{vmatrix} 4-\lambda & -2 \\ 1 & 1-\lambda \end{vmatrix} = 0$ for λ (lambda). In doing so, you are finding the <u>eigenvalues</u> of the matrix $\begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix}$.

<u>SECTION 8.5: APPLICATIONS OF DETERMINANTS</u>

<u>(THESE ARE ALL ADDITIONAL PROBLEMS)</u>

1) Use Cramer's Rule to solve the system $\begin{cases} 4x + 2y = -3 \\ x + \ \ y = -2 \end{cases}$ in Section 8.1, Exercise 3. (A)

2) Use Cramer's Rule to solve the system $\begin{cases} 3x_1 - \ \ x_2 = 15 \\ x_1 + 2x_2 = -2 \end{cases}$ in Sec. 8.3, Exercise 3. (A)

3) Find the area of the parallelogram determined by each of the following pairs of position vectors in the *xy*-plane. Distances and lengths are measured in meters. (B)
   a) $\langle 4, 0 \rangle$ and $\langle 0, 5 \rangle$
   b) $\langle 2, 5 \rangle$ and $\langle 7, 3 \rangle$
   c) $\langle 1, 4 \rangle$ and $\langle 3, 12 \rangle$. (What does the result imply about the vectors?)

4) Find the area of the triangle with vertices $(-2, -1)$, $(3, 1)$, and $(1, 5)$ in the *xy*-plane. Distances and lengths are measured in meters. (B)

CHAPTER 9:
Discrete Mathematics

(A) means "refer to Part A," (B) means "refer to Part B," etc.
Most of these exercises can be done without a calculator, though one may be permitted.

SECTION 9.1: SEQUENCES AND SERIES, and SECTION 9.6: COUNTING PRINCIPLES

1) Let $a_n = n^2 + n$. Write a_1, a_2, and a_3. (A)

2) Let $a_n = (-1)^n (2n)$. Write a_1, a_2, a_3, and a_4. (A-C)

3) Let $a_n = (-1)^{n-1} (2n-1)$. Write a_1, a_2, a_3, and a_4. (A-C)

4) Evaluate 6!. (D)

5) How many ways are there to order five tasks on a "To Do" list? (D, E)

6) Seven of your friends are sitting in a room. You have two identical plane tickets that you will give to two of them. In how many ways can this be done? (D, E)

7) Ten basketball players are to be divided into two teams of five people each. One team will be called "Team A," and the other team will be called "Team 1." In how many ways can the players be assigned to the teams? We do not yet care about positions on the teams. (D, E)

8) Simplify the expressions. Assume that n is an integer such that $n \geq 2$. (D, E)

   a) $(n+1)(n!)$; b) $\dfrac{(n+2)!}{n!}$; c) $\dfrac{(n-1)!}{(n+1)!}$; d) $\dfrac{(3n-2)!}{(3n+3)!}$ (do not multiply out)

9) Consider the sequence recursively defined by: $\begin{cases} a_1 = 4 \\ a_{k+1} = a_k + 10 \quad (k \in \mathbb{Z},\ k \geq 1) \end{cases}$.

   Find a_1, a_2, a_3, and a_4. This sequence is an <u>arithmetic sequence</u>, which we will discuss further in Section 9.2. (F)

10) Consider the sequence recursively defined by: $\begin{cases} a_1 = 2 \\ a_{k+1} = \dfrac{1}{2} a_k \quad (k \in \mathbb{Z},\ k \geq 1) \end{cases}$.

   Find a_1, a_2, a_3, and a_4. This sequence is a <u>geometric sequence</u>, which we will discuss further in Section 9.3. (F)

11) Consider the sequence recursively defined by: $\begin{cases} a_1 = -1 \\ a_{k+1} = 3a_k - 2 \quad \left(k \in \mathbb{Z},\ k \geq 1\right) \end{cases}$.

Find a_1, a_2, a_3, and a_4. (F)

12) Consider the sequence recursively defined by: $\begin{cases} a_1 = 2 \\ a_2 = 3 \\ a_{k+2} = a_{k+1}a_k \quad \left(k \in \mathbb{Z},\ k \geq 1\right) \end{cases}$.

Find a_1, a_2, a_3, a_4, and a_5. (F)

13) Evaluate $\sum_{k=1}^{4} k$. (G)

14) Evaluate $\sum_{j=3}^{7} \left(j^2 + 1\right)$. (G)

15) Evaluate $\sum_{i=2}^{4} \frac{(-1)^i}{i}$. ($i$ is not the imaginary unit here.) (G)

16) Find S_4, the fourth partial sum of the series $\sum_{k=1}^{\infty} a_k$, where $a_k = 3^k$. (G, H)

17) Write a nonrecursive expression (formula) for the apparent general n^{th} term, a_n, for each of the following sequences. Let a_1 be the initial term; i.e., assume that n begins with 1. (A-D)

a) 7, 8, 9, 10, 11, …

b) 5, 10, 15, 20, 25, …

c) 4, 7, 10, 13, 16, …

d) $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \ldots$

e) $7, \frac{7}{4}, \frac{7}{9}, \frac{7}{16}, \frac{7}{25}, \ldots$

f) $\frac{2}{3}, -\frac{4}{5}, \frac{6}{7}, -\frac{8}{9}, \frac{10}{11}, \ldots$

g) $-2, 4, -8, 16, -32, \ldots$

18) Express the apparent series using summation notation. Use k as the index of summation. (A-D, G, H)

a) $3+6+9+12+15+18$; this is a finite series.

b) $\frac{1}{4} - \frac{1}{16} + \frac{1}{64} - \frac{1}{256} + \frac{1}{1024} - \ldots$; this is an infinite series.

SECTION 9.2: ARITHMETIC SEQUENCES and PARTIAL SUMS

1) Consider the arithmetic sequence: -5, 1, 7, 13, 19, …. (A)
   a) What is the initial term, *a*?
   b) What is the common difference, *d*?

2) Consider the arithmetic sequence: $\frac{3}{2}$, 1, $\frac{1}{2}$, 0, $-\frac{1}{2}$, …. (A)
   a) What is the initial term, *a*?
   b) What is the common difference, *d*?

3) Consider the arithmetic sequence with initial term 7 and common difference 3. Assume that the initial term is a_1. (A, B)
   a) Write the first four terms of this sequence.
   b) Find S_4, the fourth partial sum.
   c) Find a_{60}, the 60th term of this sequence.

4) Consider the arithmetic sequence: 2, -3, -8, -13, -18, …. (A, B)
   a) Write a simplified, nonrecursive expression (formula) for the general n^{th} term, a_n, for this sequence. Let a_1 be the initial term.
   b) Use a) to find a_{387}. (Calculator)

5) An arithmetic sequence has $a_1 = 6$ and $a_{200} = 1399$. Find a_{123}. (Calculator)

SECTION 9.3: GEOMETRIC SEQUENCES, PARTIAL SUMS, and SERIES

1) Consider the geometric sequence: 4, 20, 100, 500, 2500, …. (A)
   a) What is the initial term, *a*?
   b) What is the common ratio, *r*?

2) Consider the geometric sequence: $\frac{6}{7}$, $-\frac{2}{7}$, $\frac{2}{21}$, $-\frac{2}{63}$, $\frac{2}{189}$ …. (A)
   a) What is the initial term, *a*?
   b) What is the common ratio, *r*?

3) Consider the geometric sequence with initial term 5 and common ratio -4. Assume that the initial term is a_1. (A, B)
   a) Write the first four terms of this sequence.
   b) Find S_4, the fourth partial sum.
   c) Find a_{12}, the 12th term of this sequence. (Calculator)
   d) As $n \to \infty$, do the terms of the sequence approach a real number? If so, what number?

4) Consider the geometric sequence: $\frac{2}{9}, \frac{1}{3}, \frac{1}{2}, \frac{3}{4}, \frac{9}{8}, \ldots$. (A, B)
   a) Write a simplified, nonrecursive expression (formula) for the general n^{th} term, a_n, for this sequence. Let a_1 be the initial term.
   b) Use a) to find a_{10}. (Calculator)
   c) Verify your answer to b) by recursively using the common ratio to find a_6, a_7, a_8, a_9, and a_{10}.
   d) As $n \to \infty$, do the terms of the sequence approach a real number? If so, what number?

5) A geometric sequence has $a_1 = 3$ and $a_4 = -\frac{24}{125}$. (A, B)
   a) Find a_2 and a_3.
   b) As $n \to \infty$, do the terms of the sequence approach a real number? If so, what number?

6) Let $\{a_n\}$ be the sequence from Exercise 3. Does the series $\sum_{n=1}^{\infty} a_n$ converge or diverge? If the series converges, find its sum. (D)

7) Let $\{a_n\}$ be the sequence from Exercise 4. Does the series $\sum_{n=1}^{\infty} a_n$ converge or diverge? If the series converges, find its sum. (D)

8) Let $\{a_n\}$ be the sequence from Exercise 5. Does the series $\sum_{n=1}^{\infty} a_n$ converge or diverge? If the series converges, find its sum. (D)

9) For what values of x does the series $\sum_{n=1}^{\infty} (3x)^n$ converge? (D)

10) Consider the infinite geometric series: $\frac{2}{3}+2+6+18+54+....$ (D)

   a) Write the series using summation notation.
   b) Is the series convergent or divergent?
   c) If the series is convergent, find its sum. If the series is divergent, write "No sum."

11) Consider the infinite geometric series: $-3+3-3+3-3+....$ (D)
   Do a), b), and c) from 10) for this series.

12) Consider the infinite geometric series: $5-\frac{5}{4}+\frac{5}{16}-\frac{5}{64}+\frac{5}{256}-....$ (D)
   Do a), b), and c) from 10) for this series.

13) Write $0.\overline{39}$ as a nice (simplified) fraction of the form $\frac{\text{integer}}{\text{integer}}$. (D)

14) **ADDITIONAL PROBLEM:** Write $0.5\overline{172}$ as a fraction of the form $\frac{\text{integer}}{\text{integer}}$. (D)

SECTION 9.4: MATHEMATICAL INDUCTION

1) Prove using mathematical induction: $1+2+3+...+n=\frac{n(n+1)}{2}$ $\left(\forall n \in \mathbb{Z}^+\right)$,
   where $\mathbb{Z}^+$ is the set of positive integers.

2) Use the statement proven in 1) to evaluate $\sum_{k=1}^{1000} k$.

SECTION 9.5: THE BINOMIAL THEOREM

1) Expand and simplify $(x+y)^5$ using the Binomial Theorem. (A, B)
2) Expand and simplify $(a+b)^6$ using the Binomial Theorem. (A, B)
3) Expand and simplify $(2x+3y)^3$ using the Binomial Theorem. (A-C)
4) Expand and simplify $(a-2b)^4$ using the Binomial Theorem. (A-C)
5) Let $f(x)=x^3$. Simplify the difference quotient completely: (A, B, Section 1.10)

$$\frac{f(x+h)-f(x)}{h} \quad (h \neq 0)$$

ADDITIONAL PROBLEM: Find $f'(x)$ by letting $h \to 0$. (Section 1.11)

CHAPTER 10:
Conics and Polar Coordinates

(A) means "refer to Part A," (B) means "refer to Part B," etc.
Most of these exercises can be done without a calculator, though one may be permitted.

SECTION 10.3: ELLIPSES

1) An ellipse has equation $4x^2 + 25y^2 - 24x + 200y + 336 = 0$ in the usual *xy*-plane. (B-F)
   a) Find the standard form of the equation of this ellipse.
   b) The center of this ellipse is at what point?
   c) The vertices of this ellipse are at what points?
   d) The foci of this ellipse are at what points?
   e) What is the eccentricity of this ellipse?

2) Repeat Exercise 1 for the equation $16x^2 + 9y^2 + 160x + 18y + 265 = 0$. (B-F)

SECTION 10.4: HYPERBOLAS

Know the graphs of $x^2 - y^2 = 1$ and $y^2 - x^2 = 1$.

1) **ADDITIONAL PROBLEM:** Sketch the graph of $\frac{x^2}{4} - \frac{y^2}{9} = 1$.

2) **ADDITIONAL PROBLEM:** Sketch the graph of $\frac{(y-2)^2}{1} - \frac{(x-1)^2}{4} = 1$.

SECTION 10.8: POLAR COORDINATES

1) Sketch the graph of the polar equation $r = 2\sin\theta$. (A, B)

2) Sketch the graph of the polar equation $r = 3\cos\theta$. (A, B)
   ADDITIONAL PROBLEM: Write the corresponding Cartesian (or rectangular) equation in *x* and *y*. Hint: First multiply both sides by *r*, with the understanding that $r = 0$ corresponds to the pole (the origin in Cartesian coordinates). (C)

POLAR GRAPH PAPER