CHAPTER 0:

Preliminary Topics

SECTION 0.1: SETS OF NUMBERS

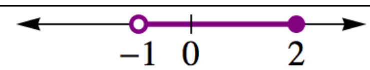
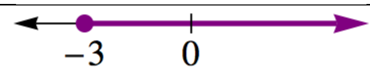
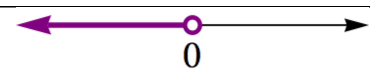
1) -12 is an integer. -12, $\frac{5}{7}$, and $7.\overline{13}$ are rational numbers. All the listed numbers are real numbers.

2) -12 is an integer, or -12 is in the set of integers. True.

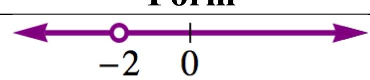
3) π is not in the set of rational numbers. (Since π is real, that means that π is an irrational number.) True.

4) $\ni$ denotes "such that." $\forall$ denotes "for all" or "for every." $\exists$ denotes "there exists."

5)

Set-builder Form	Graphical Form	Interval Form
$\{x \in \mathbb{R} \mid -1 < x \le 2\}$		$(-1, 2]$
$\{x \in \mathbb{R} \mid x \ge -3\}$		$[-3, \infty)$
$\{x \in \mathbb{R} \mid x < 0\}$		$(-\infty, 0)$

6)

Set-builder Form	Graphical Form	Interval Form
$\{x \in \mathbb{R} \mid x \ne -2\}$		$(-\infty, -2) \cup (-2, \infty)$

7) a) $(2, 4]$; b) $[3, 9]$

8) If $T \in T$, then, by the definition of *T*, $T \notin T$, which is a contradiction.
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SECTION 0.2: LOGIC

1)

a) If Arnold is an American, then Arnold is a Californian. False; Arnold could be an Alabaman (for example).

b) If Arnold is not a Californian, then Arnold is not an American. False; Arnold could be an Alabaman (for example).

c) If Arnold is not an American, then Arnold is not a Californian. True.

d) Its contrapositive

2) Yes

3) No

4) False. The following is true: $x^2 = 9 \Leftrightarrow (x = -3 \text{ or } x = 3)$.

SECTION 0.3: ROUNDING

1) a) 42.4; b) 42.40; c) 42.397; d) 42.3972

2)

Decimal	**Number of Decimal Places**	**Number of Significant Digits (or Figures)**
7.583	3	4
0.00510	5	3
600.0020	4	7

SECTION 0.4: ABSOLUTE VALUE AND DISTANCE

1) a) π; b) 0; c) 7.5

2)

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

3) $|x - a|$, or $|a - x|$

SECTION 0.5: EXPONENTS AND RADICALS: LAWS AND FORMS

1) a) 7; b) not real; c) $6\sqrt{2}$; d) -4; e) $\sqrt{6}+\sqrt{2}$ (not equivalent to $\sqrt{8}$)

2) a) $\left| x+4 \right|$; b) $x+4$

3) Counterexample: $\sqrt{1+4} \neq \sqrt{1}+\sqrt{4}$, because $\sqrt{5} \neq 3$.

4) a) $3y^{-1}$; b) $\dfrac{4}{5}x^{-1/3}$

5)
   a) $\dfrac{a^9}{125}$ $(a \neq 0)$

   b) $x^{37/12}$ $(x > 0)$; $x^{37/12} = \sqrt[12]{x^{37}}$, or $\left(\sqrt[12]{x}\right)^{37}$, or $x^3\left(\sqrt[12]{x}\right)$

   c) 5^{9n}, or $1{,}953{,}125^n$

SECTION 0.6: POLYNOMIAL, RATIONAL, AND ALGEBRAIC EXPRESSIONS

1) a) 4; b) 7; c) $7x^4$; d) $a_0 = -\pi$, $a_1 = 1$, $a_2 = -\dfrac{1}{3}$, $a_3 = 0$, $a_4 = 7$

2) $6x^2 - x$, or $-x + 6x^2$

3) $9x^2 - 24x + 16$

4) No, it is not a polynomial in x, because a polynomial in x cannot have negative exponents on x. Yes, it is a rational expression in x, because $4x^{-2} = \dfrac{4}{x^2}$, a quotient of nonzero polynomials.

5) No, it is not a rational expression in x. Yes, it is an algebraic expression in x.

SECTION 0.7: FACTORING POLYNOMIALS

1)

a) $5x^6\left(3x^4+5\right)$

b) $2x^5\left(x+4\right)\left(x-4\right)$

c) Prime

d) $\left(a^2+1\right)\left(a+1\right)\left(a-1\right)$

e) $-4\left(x+5\right)\left(x+3\right)$

f) $\left(2x-3\right)^2$. The discriminant is 0, so the trinomial is a PST.

g) $\left(3x-1\right)\left(2x+3\right)$

h) Prime. The discriminant is -20, so the trinomial is prime.

i) $\left(x^2-3\right)\left(x+4\right)$

j) $\left(2x+y\right)\left(4x^2-2xy+y^2\right)$

k) $\left(2x-y\right)\left(4x^2+2xy+y^2\right)$

l) $2a\left(a+4\right)\left(a^2-4a+16\right)$

2) No. Exponents must be applied to all factors of the base.

3) $\left(x^2+3\right)\left(x^2-2\right)$

SECTION 0.8: FACTORING RATIONAL AND ALGEBRAIC EXPRESSIONS

1)

a) $\dfrac{2\left(x^2+1\right)^2}{x^5}$

b) $x^{1/2}\left(1+x\right)\left(1-x\right)$, or $\sqrt{x}\left(1+x\right)\left(1-x\right)$, or $\sqrt{x}\left(1+x\right)\left(1+\sqrt{x}\right)\left(1-\sqrt{x}\right)$, or $-x^{1/2}\left(x+1\right)\left(x-1\right)$

c) $\dfrac{2\left(2x^2+6x+5\right)}{\left(2x+3\right)^{3/4}}$

SECTION 0.9: SIMPLIFYING ALGEBRAIC EXPRESSIONS

1)

a) $\dfrac{2}{3a^2+4a}$, or $\dfrac{2}{a\left(3a+4\right)}$

b) $t^2-3t+9 \ \left(t\neq -3\right)$

c) $-\dfrac{4+x}{x+3}\ \left(x\neq 4\right)$, or $-\dfrac{x+4}{x+3}\ \left(x\neq 4\right)$

d) $\dfrac{2-30x^2}{\left(5x^2+1\right)^3}$, or $\dfrac{2\left(1-15x^2\right)}{\left(5x^2+1\right)^3}$, or $-\dfrac{2\left(15x^2-1\right)}{\left(5x^2+1\right)^3}$

e) $\dfrac{50r^3-45r^2}{\left(4r-3\right)^{3/2}}$, or $\dfrac{5r^2\left(10r-9\right)}{\left(4r-3\right)^{3/2}}$

f) $\dfrac{12x^2+40x}{\left(2x^3+5\right)^{5/3}}$, or $\dfrac{4x\left(3x+10\right)}{\left(2x^3+5\right)^{5/3}}$

g) $-\dfrac{1}{x-2}\ \left(x<3\right)$, or $\dfrac{1}{2-x}\ \left(x<3\right)$

2) $\dfrac{\sqrt{x}-\sqrt{a}}{x-a}$. Notes: The given expression requires: $x \ge 0$, $a \ge 0$, and $(x \ne 0 \text{ or } a \ne 0)$. Furthermore and more precisely:

$$\frac{1}{\sqrt{x}+\sqrt{a}} = \begin{cases} \dfrac{\sqrt{x}-\sqrt{a}}{x-a}, & \text{if } x \ne a \\ \dfrac{1}{2\sqrt{a}} = \dfrac{\sqrt{a}}{2a}, & \text{if } x = a \end{cases}$$

3) $-\dfrac{1}{4+\sqrt{9+x}}$ $(x \ne 7)$

4) "Yes": b), c), and i). The others are "No."

SECTION 0.10: MORE ALGEBRAIC MANIPULATIONS

1)

$$\frac{7\sqrt{x}-x^2+4x^5+12x^9}{6x^5} = \boxed{\frac{7}{6}}x^{\boxed{-\frac{9}{2}}} - \boxed{\frac{1}{6}}x^{\boxed{-3}} + \boxed{\frac{2}{3}} + \boxed{2}x^{\boxed{4}}$$

2)

$$\frac{4x^2}{9} - 16y^2 = \frac{x^2}{\boxed{\frac{9}{4}}} - \frac{y^2}{\boxed{\frac{1}{16}}}$$

3)

$$\frac{x^2}{x^2+9} = 1 - \boxed{\frac{9}{x^2+9}}$$

4)

$$(12x-7)^3 = \boxed{\frac{1}{12}}(12x-7)^3(12)$$

SECTION 0.11: SOLVING EQUATIONS

1) a) $\{5\}$; b) $\mathbb{R}$; c) $\varnothing$

2)

a) $\{-2, 6\}$. Hint: Rewrite as $x^2 - 4x - 12 = 0 \Leftrightarrow \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-12)}}{2(1)}$.

b) $\{-2, 6\}$. Rewrite as: $x^2 - 4x - 12 = 0 \Leftrightarrow (x-6)(x+2) = 0$.

c) $\{-2, 6\}$. Rewrite as: $x^2 - 4x + 4 = 16 \Leftrightarrow (x-2)^2 = 16 \Leftrightarrow x - 2 = \pm 4$.

3) $\{-3\sqrt{3}, 3\sqrt{3}\}$. (The solutions are $\pm 3\sqrt{3}$.)

4) $\left\{\dfrac{-3-2\sqrt{6}}{3}, \dfrac{-3+2\sqrt{6}}{3}\right\}$. (The solutions are $\dfrac{-3 \pm 2\sqrt{6}}{3}$.)

5) $\{2\}$

6) $\{2\}$. (We must reject -1.)

7) $\varnothing$. (We must reject -8.)

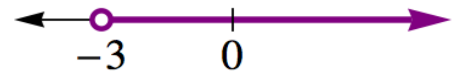
8) $\{-7, -1\}$. The two solutions are 3 units away from -4 on the real number line.
$|x+4| = |x-(-4)|$, the distance between x and -4, which is 3 for those solutions.

9) $\varnothing$

SECTION 0.12: SOLVING INEQUALITIES

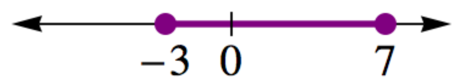
1)

a) The solution set …

… in set-builder form is:	$\{x \in \mathbb{R} \mid x > -3\}$, or $\{x \in \mathbb{R} : x > -3\}$
… in graphical form is:	
… in interval form is:	$(-3, \infty)$

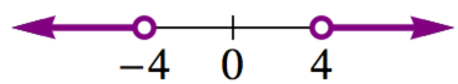
b) $\varnothing$

c) The solution set …

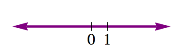
… in set-builder form is:	$\{x \in \mathbb{R} \mid -3 \le x \le 7\}$, or $\{x \in \mathbb{R} : -3 \le x \le 7\}$
… in graphical form is:	
… in interval form is:	$[-3, 7]$

The solutions are no more than 5 units away from 2 on the real number line.

d) The solution set …

… in set-builder form is:	$\{x \in \mathbb{R} \mid x < -4 \text{ or } x > 4\}$, or $\{x \in \mathbb{R} : x < -4 \text{ or } x > 4\}$
… in graphical form is:	
… in interval form is:	$(-\infty, -4) \cup (4, \infty)$

The solutions are further than 4 units away from 0 on the real number line.

e) The solution set is $\mathbb{R}$. Graphically, it's the entire real number line:  .
In interval form, it's $(-\infty, \infty)$. Note: Absolute value is never negative.

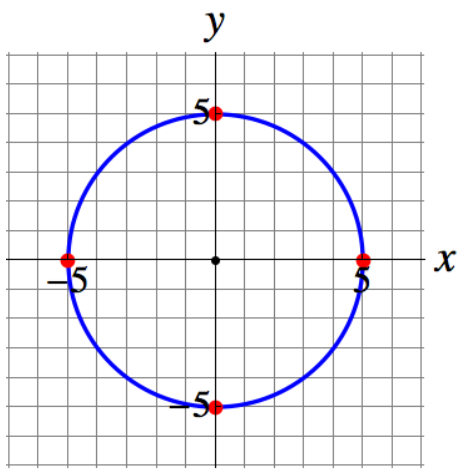
SECTION 0.13: THE CARTESIAN PLANE and CIRCLES

1)

a) $3\sqrt{5}$ meters, or $3\sqrt{5}$ m

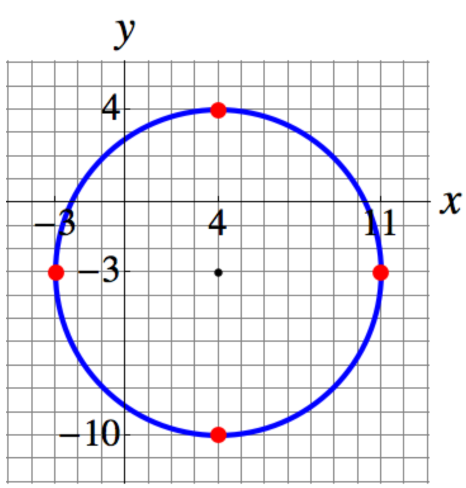
b) $\left(-\frac{7}{2}, 2\right)$, or $(-3.5, 2)$

2) $x^2 + y^2 = 25$. (Note: The center of the circle is not a point on the circle.)

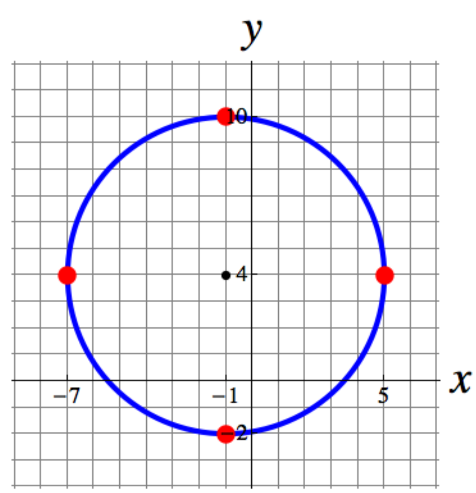


3) $(x-4)^2 + (y+3)^2 = 49$.

Graph for Exercise 3



Graph for Exercise 4

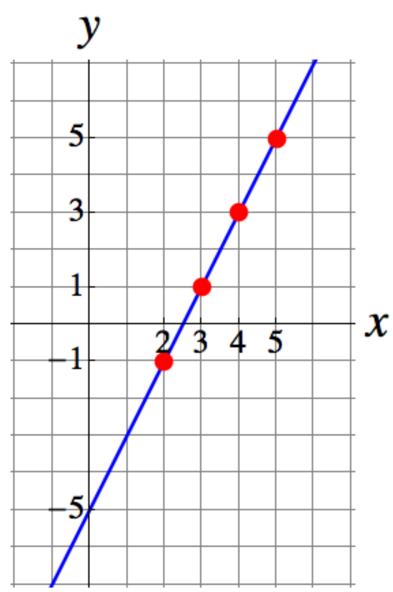


4) $(x+1)^2 + (y-4)^2 = 36$. The center is $(-1, 4)$, and the radius is 6.

5) $\left(x+\frac{7}{2}\right)^2 + (y-2)^2 = \frac{45}{4}$, or $(x+3.5)^2 + (y-2)^2 = 11.25$

SECTION 0.14: LINES

1) Some possible answers: $(4,3)$, $(5,5)$, $(2,-1)$



2) $y=1$

3) $x=3$

4) $y-1=-\frac{1}{2}(x-3)$, or $y-(-1)=-\frac{1}{2}(x-7)$

5) $y=-\frac{1}{2}x+\frac{5}{2}$. The y-intercept is $\frac{5}{2}$, or $\left(0,\frac{5}{2}\right)$.

6) $y=\frac{2}{9}x+\frac{26}{9}$

7) Parallel: $\frac{3}{4}$; Perpendicular: $-\frac{4}{3}$

8) $y-(-3)=-\frac{1}{7}(x-(-2))$

9) a) 6, or $(6,0)$; b) 8, or $(0,8)$; c) $\frac{x}{6}+\frac{y}{8}=1$

SECTION 0.15: PLANE AND SOLID GEOMETRY

1) Let b_1 and b_2 be the bases of the two triangles. Their common height is h, the height of the trapezoid. The area of the trapezoid is: $\frac{1}{2}b_1h + \frac{1}{2}b_2h = \left(\frac{b_1 + b_2}{2}\right)h$.

2)
   a) 75π cubic inches, or $75\pi\ \text{in}^3$
   b) 30π square inches, or $30\pi\ \text{in}^2$
   c) 80π square inches, or $80\pi\ \text{in}^2$
   d) 25π cubic inches, or $25\pi\ \text{in}^3$

3)
   a) Exact: $\sqrt[3]{\frac{75}{\pi}}$ meters, or $\sqrt[3]{\frac{75}{\pi}}$ m. Approximate: 2.879 meters, or 2.879 m.
   b) 104.2 square meters, or $104.2\ \text{m}^2$

SECTION 0.16: VARIATION

1) $a = 4b\sqrt{c}$

2) $w = \frac{7m^2}{200n^3}$

3) directly proportional