CHAPTER 1:

Functions

SECTION 1.1: FUNCTIONS

1) $f(x) = -x$
2) $f(x) = |x|$
3)

Input x	Output $f(x)$
-1	3
0	4
1	5
2	6
$\sqrt{5}$	$\sqrt{5}+4$
π	$\pi+4$
10/3	22/3
4.7	8.7
c	$c+4$
$a+h$	$a+h+4$

4)

Input x	Output $f(x)$
-1	-2
0	0
1	2
2	4
$\sqrt{5}$	$2\sqrt{5}$
π	2π
10/3	20/3
4.7	9.4
c	2c
$a+h$	$2(a+h)$, or $2a+2h$

5)

Input t	Output $g(t)$
-2	27
-1	15
0	5
1	-3
2	-9
c	c^2-9c+5

6)

Input r	Output $h(r)$
-2	-7
-1	8
0	9
1	8
2	-7
c	$9-c^4$

7)

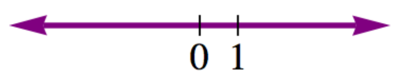
Input r	Output $V(r)$
1	$\frac{4\pi}{3}$
2	$\frac{32\pi}{3}$
5/2	$\frac{125\pi}{6}$
c	$\frac{4}{3}\pi c^3$

$V(r)$ is the volume of a sphere of radius r.

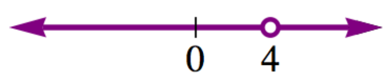
8)

Input x	Output $f(x)$
1	7
2	7
5/2	7
c	7

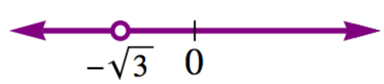
9)

a)	$\mathbb{R}$
b) graphical form	
c) interval form	$(-\infty, \infty)$

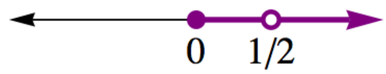
10)

a) set-builder form	$\{x \in \mathbb{R} \mid x \neq 4\}$, or $\{x \in \mathbb{R} : x \neq 4\}$
b) graphical form	
c) interval form	$(-\infty, 4) \cup (4, \infty)$

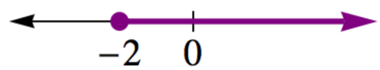
11)

a) set-builder form	$\{t \in \mathbb{R} \mid t \neq -\sqrt{3}\}$, or $\{t \in \mathbb{R} : t \neq -\sqrt{3}\}$
b) graphical form	
c) interval form	$(-\infty, -\sqrt{3}) \cup (-\sqrt{3}, \infty)$

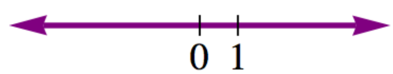
12)

a) set-builder form	$\left\{ t \in \mathbb{R} \;\middle|\; t \geq 0 \text{ and } t \neq \frac{1}{2} \right\}$, or $\left\{ t \in \mathbb{R} : t \geq 0 \text{ and } t \neq \frac{1}{2} \right\}$
b) graphical form	
c) interval form	$\left[0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

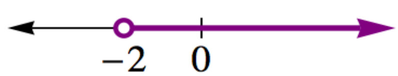
13)

a) set-builder form	$\{ r \in \mathbb{R} \mid r \geq -2 \}$, or $\{ r \in \mathbb{R} : r \geq -2 \}$
b) graphical form	
c) interval form	$[-2, \infty)$

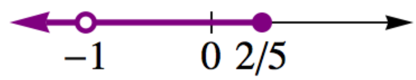
14)

a)	$\mathbb{R}$
b) graphical form	
c) interval form	$(-\infty, \infty)$

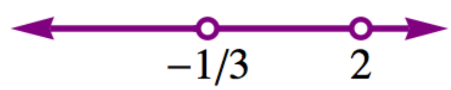
15)

a) set-builder form	$\{ r \in \mathbb{R} \mid r > -2 \}$, or $\{ r \in \mathbb{R} : r > -2 \}$
b) graphical form	
c) interval form	$(-2, \infty)$

16)

a) set-builder form	$\left\{x \in \mathbb{R} \,\middle\vert\, x \leq \frac{2}{5} \text{ and } x \neq -1\right\}$, or $\left\{x \in \mathbb{R} : x \leq \frac{2}{5} \text{ and } x \neq -1\right\}$
b) graphical form	
c) interval form	$(-\infty, -1) \cup \left(-1, \frac{2}{5}\right]$

17)

a) set-builder form	$\left\{t \in \mathbb{R} \,\middle\vert\, t \neq -\frac{1}{3} \text{ and } t \neq 2\right\}$, or $\left\{t \in \mathbb{R} : t \neq -\frac{1}{3} \text{ and } t \neq 2\right\}$
b) graphical form	
c) interval form	$\left(-\infty, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, 2\right) \cup (2, \infty)$

18) a) 2, b) $\sqrt{11}$, c) $\sqrt{a}$, d) $\sqrt{x+h}$, e) $|x+1|$

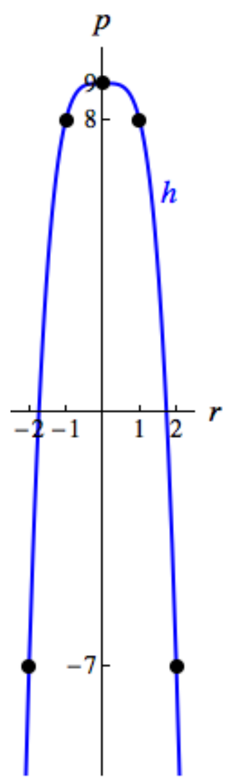
19) a) 2, b) $2x^2 - 3x - 7$, c) $2t^4 - 3t^2 - 7$, d) $2t^2 + 4th + 2h^2 - 3t - 3h - 7$

20) a) $\frac{3}{11}$, b) $\frac{3}{2\pi + 1}$, c) $\frac{3}{2x + 2h + 1}$

21) 3

SECTION 1.2: GRAPHS OF FUNCTIONS

1)



2) a) Yes, b) No

3) Domain: $(-\infty, \infty)$. Range: $(-\infty, 9]$.

4) a) 6; b) 8; c) -2, 1, and 3

5)

a) x-intercept: 4, or $(4, 0)$; y-intercept: -2, or $(0, -2)$

b) x-intercepts: NONE; y-intercept: 3, or $(0, 3)$

c) x-intercepts: $\dfrac{-3 \pm 3\sqrt{17}}{8}$, or $\dfrac{3}{8}\left(-1 \pm \sqrt{17}\right)$, or $\left(\dfrac{-3 \pm 3\sqrt{17}}{8}, 0\right)$;

y-intercept: 3, or $(0, 3)$

d) x-intercept: $\dfrac{1}{5}$, or $\left(\dfrac{1}{5}, 0\right)$; y-intercept: -1, or $(0, -1)$

6) Interval of increase: $(-\infty, 0]$. Interval of decrease: $[0, \infty)$.

7)

a) The graph passes the Vertical Line Test (VLT). The coin cannot be at two different heights at the same time.

b) $[0, 4]$ in seconds

c) $[0, 256]$ in feet

d) 256 feet. Note: This is $s(0)$, the initial height of the coin.

e) 4 seconds

f) The earth's gravitational pull is causing the coin to drop faster and faster until it hits the ground.

g) 2.5 seconds after the coin was dropped, it was 156 feet above the ground.

h) 240 feet

i) Exactly $\frac{\sqrt{39}}{2}$ seconds. Approximately 3.12 seconds.

j) Hint: Solve $s(t) = 0$, and reject the negative solution as extraneous.

SECTION 1.3: BASIC GRAPHS and SYMMETRY

1)

a) even; show that $f(-x) = f(x),\ \forall x \in \mathbb{R}$

b) the y-axis

2)

a) odd; show that $g(-t) = -g(t),\ \forall t \in \mathbb{R}$

b) the origin

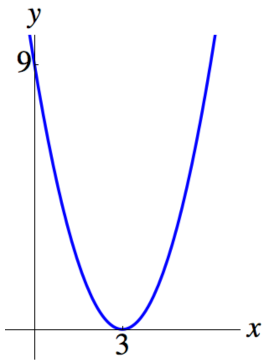
3) No, s is not even, because $\text{Dom}(s)$ is not symmetric about 0.

4) If f is a function, the graph of $y = f(x)$ passes the Vertical Line Test (VLT). Only in the case of a zero function, where $f(x) = 0$ on a domain that is a nonempty subset of $\mathbb{R}$, will the graph be symmetric about the x-axis.

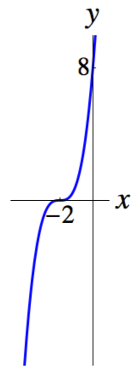
SECTION 1.4: TRANSFORMATIONS

1)

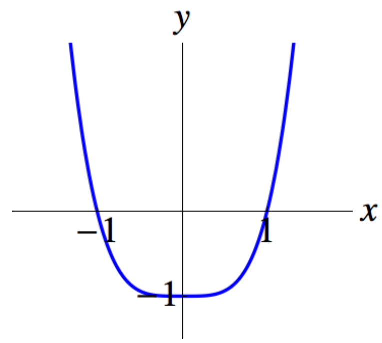
a)



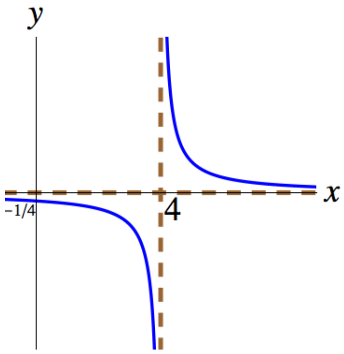
b)



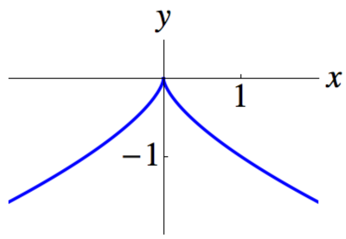
c)



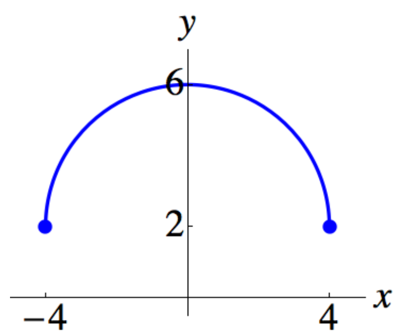
d)



e)

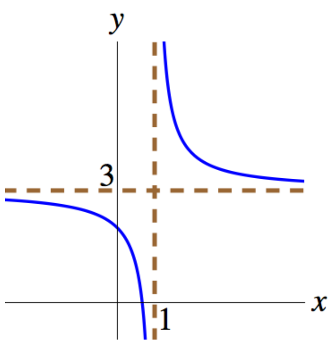


f)

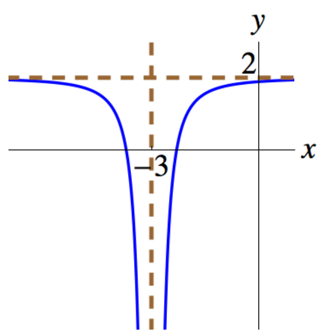


2)

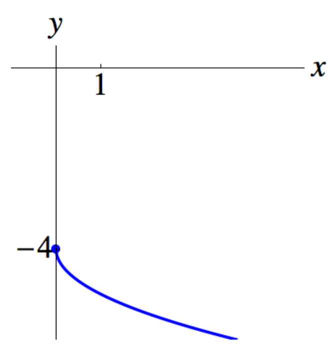
a)



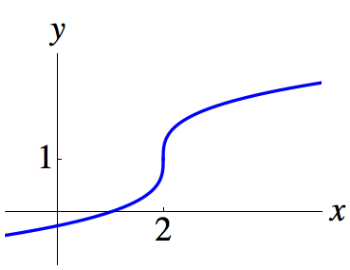
b)



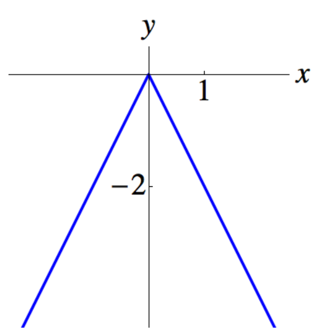
c)



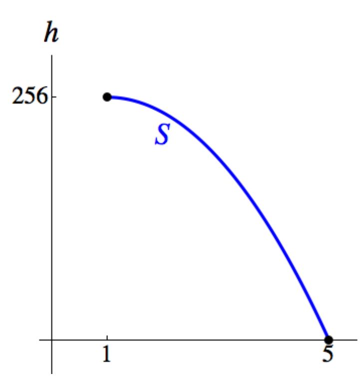
d)



e)



3)



3) $S(t) = -16(t-1)^2 + 256$, where $\text{Dom}(S) = [1, 5]$ in seconds. See preceding graph.

4) a) $y = \sqrt{x+2} + 3$, b) $y = -(x-2)^2 - 1$

5) a) $(-6, -3)$, b) $(-1, 8)$, c) $(-4, -2)$, d) $(4, 2)$, e) $(-4, 6)$

6)

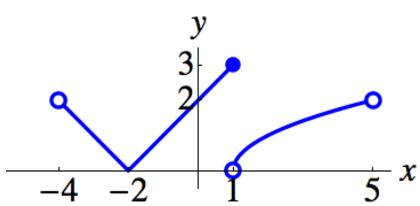
a) Its only zeros are 5 and 11.
b) 3 and 9 are not zeros. (Remember the VLT.)
c) Its only zeros are 1 and 3.
d) Its only zeros are 3 and 9.
e) Its only zeros are 3 and 9.
f) Its only zeros are -3 and -9.

SECTION 1.5: PIECEWISE-DEFINED FUNCTIONS; LIMITS AND CONTINUITY IN CALCULUS

1)

a) 1, b) 3, c) $\sqrt{2}$, d) undefined

e)

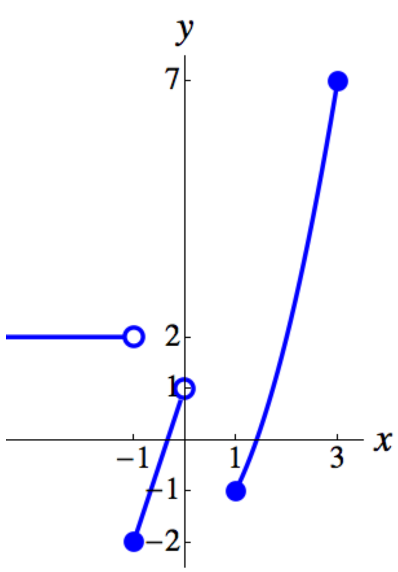


f) $(-4, 5)$, g) $[0, 3]$

2)

a) 2, b) -2, c) undefined, d) 7

e)



f) $(-\infty, 0) \cup [1, 3]$, g) $[-2, 7]$

3) a) 2, b) 5, c) -4, d) -7

SECTION 1.6: COMBINING FUNCTIONS

1)

a) $2\sqrt{x}-4$; $[0,\infty)$

b) 4; $[0,\infty)$

c) $\sqrt{x}\left(\sqrt{x}-4\right)$, or $x-4\sqrt{x}$; $[0,\infty)$

d) $\dfrac{\sqrt{x}}{\sqrt{x}-4}$, or, if we rationalize the denominator, $\dfrac{\sqrt{x}\left(\sqrt{x}+4\right)}{x-16}$, or $\dfrac{x+4\sqrt{x}}{x-16}$; $[0,16)\cup(16,\infty)$

e) $5\sqrt{x}-12$; $[0,\infty)$

f) $\sqrt{\sqrt{x}-4}$; $[16,\infty)$

g) 2

2) No, because $(3x)^2$ and $3x^2$ are not equivalent. This tells us that $f\circ g$ and $g\circ f$ are not necessarily equal (i.e., they are not necessarily the same function) for functions f and g. (Can you think of functions f and g where $f\circ g=g\circ f$?)

3)

a) $\left(\dfrac{1}{x-9}\right)^2$, or $\dfrac{1}{(x-9)^2}$; $(-\infty,9)\cup(9,\infty)$

b) $\dfrac{1}{x^2-9}$; $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

c) No

4) Answers may vary.

a) $g(x)=x^2-7$, $f(u)=\sqrt[3]{u}$

b) $g(x)=4x+3$, $f(u)=u^8$

c) $g(x)=2x-9$, $f(u)=\dfrac{5}{u}$

SECTION 1.7: SYMMETRY REVISITED

1)

a) odd; the origin
b) even; the y-axis
c) neither; neither
d) neither; neither

2)

a) even; prove that $f(-x) = f(x),\ \forall x \in \mathbb{R} \setminus \{0\}$

b) neither; for example, if $t = 1$, $f(1) = 1$, but $f(-1)$ is undefined. More generally, the domain of an even or odd function must be symmetric about 0. That is not the case here.

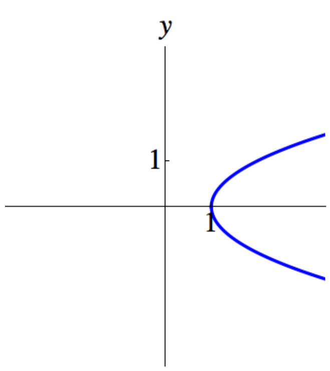
c) odd; prove that $g(-x) = -g(x),\ \forall x \in \mathbb{R} \setminus \{0\}$

d) neither; for example, if $r = 1$, $h(1) = 2$, and $h(-1) = 0$. $h(1)$ and $h(-1)$ are neither equal nor opposites.

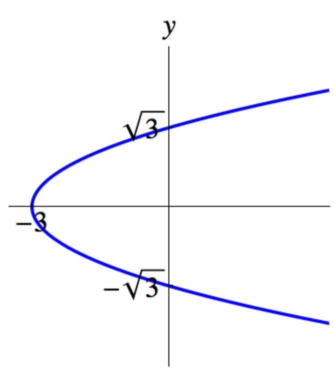
SECTION 1.8: $x = f(y)$

1)

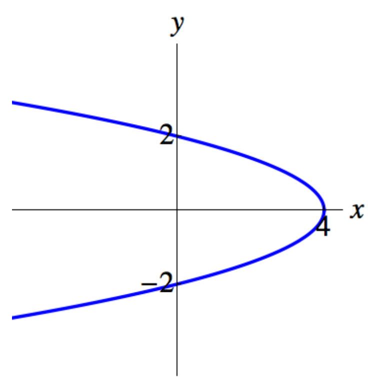
a)



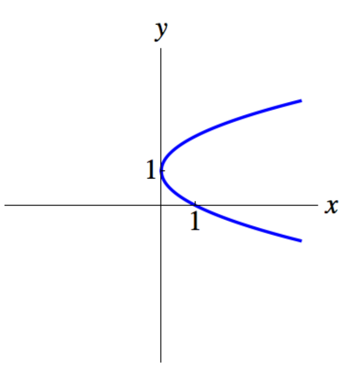
b)



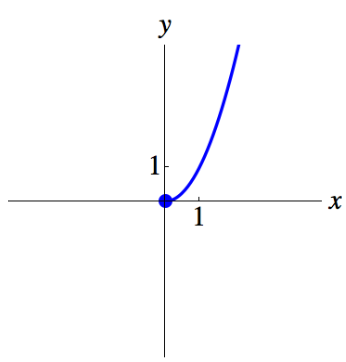
c)



d)



e)



SECTION 1.9: INVERSES OF ONE-TO-ONE FUNCTIONS

1) $f^{-1}(x) = x^5$ on $\mathbb{R}$

2)

a)

f^{-1}	
Input x	Output $f^{-1}(x)$
-8	-2
1	1
π^3	π

b) -2

c) $\{-2, 1, \pi\}$, the same as the domain of f

d) π

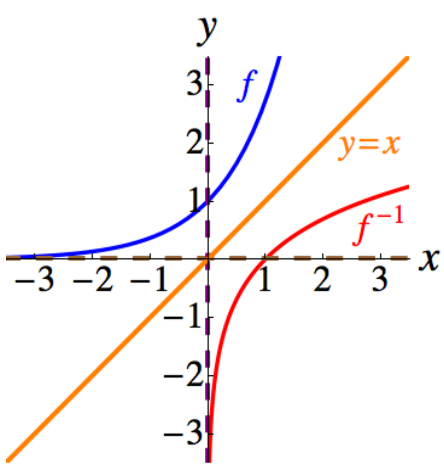
e) -8

3) a) No, b) Yes

4) Yes

5) $(2, -4)$

6)



We will revisit these functions in Chapter 3: $f(x) = e^x$ and $f^{-1}(x) = \ln x$.

7) $f^{-1}(x) = \dfrac{8\left(\sqrt[5]{x}\right) - 1}{3}$ on $\mathbb{R}$

8) $g^{-1}(x)=\dfrac{\left(\dfrac{x}{2}\right)^3+5}{4}=\dfrac{x^3+40}{32}$ (Simplified form) on $\mathbb{R}$. $g^{-1}(6)=8$.

9) a) $f(x)=\dfrac{9}{5}x+32$, b) $f^{-1}(x)=\dfrac{5}{9}(x-32)$

SECTION 1.10: DIFFERENCE QUOTIENTS

1) -13

2) $77\ \dfrac{\text{meters}}{\text{second}}\left(\text{or } \dfrac{\text{m}}{\text{s}}\right)$

3) $-64\ \dfrac{\text{feet}}{\text{second}}\left(\text{or } \dfrac{\text{ft}}{\text{s}}\right)$. This is negative, because the coin is falling (its height is decreasing).

4) $-\dfrac{1}{x(x+h)}\ (h\neq 0)$, or $-\dfrac{1}{x^2+xh}\ (h\neq 0)$

5) $\dfrac{1}{\sqrt{x+h}+\sqrt{x}}\ (h\neq 0)$

6) $10t+5h+3\ (h\neq 0)$

SECTION 1.11: LIMITS AND DERIVATIVES IN CALCULUS

1)

a) $-\dfrac{1}{x^2}$

b) $\dfrac{1}{2\sqrt{x}}$

c) $10t+3$