CHAPTER 2:

Polynomial and Rational Functions

SECTION 2.1: QUADRATIC FUNCTIONS (AND PARABOLAS)

1)

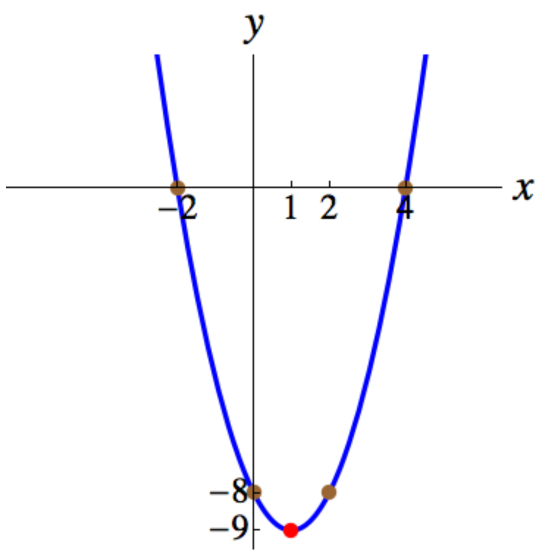
a) Upward; b) $(1,-9)$; c) $x=1$

d) -8, or $(0,-8)$

e) -2 and 4, or $(-2,0)$ and $(4,0)$

f) $(2,-8)$

g)



2)

a) Downward; b) $\left(-\frac{11}{4},\frac{243}{8}\right)$, or $(-2.75, 30.375)$; c) $x=-\frac{11}{4}$, or $x=-2.75$

d) -15, or $(0,-15)$

e) -5 and $-\frac{1}{2}$, or $(-5,0)$ and $\left(-\frac{1}{2},0\right)$

3)

a) Upward; b) $(2, 3)$; c) $x = 2$

d) 7, or $(0, 7)$

e) None

4) a) $y = (x+4)^2 - 6$; b) Upward; c) $(-4, -6)$

5) a) $y = -4(x-3)^2 - 1$; b) Downward; c) $(3, -1)$

6) a) $y = 2\left(x - \frac{7}{2}\right)^2 + \frac{5}{2}$, or $y = 2(x-3.5)^2 + 2.5$

b) Upward

c) $\left(\frac{7}{2}, \frac{5}{2}\right)$, or $(3.5, 2.5)$

7) $y = -\frac{2}{9}(x-6)^2 - 3$, or $y = -\frac{2}{9}x^2 + \frac{8}{3}x - 11$

8)

a) $-\$2000$ (i.e., a net loss of \$2000)

b) 7 widgets

c) \$450

d) 4 widgets and 10 widgets

9)

a) 384 feet

b) $\frac{5}{2}$ seconds, or 2.5 seconds

c) 484 feet

d) 8 seconds

SECTION 2.2: POLYNOMIAL FUNCTIONS OF HIGHER DEGREE

1)
   a) ∞, $-\infty$
   b) ∞, ∞
   c) $-\infty$, $-\infty$
   d) $-\infty$, ∞

2) 3 or 1

3) 6, 4, 2, or 0

4) The degree of f is odd and is at least 5. The leading coefficient of $f(x)$ is positive. $f(0)=1$. f has exactly one distinct negative real zero, and it has odd multiplicity.

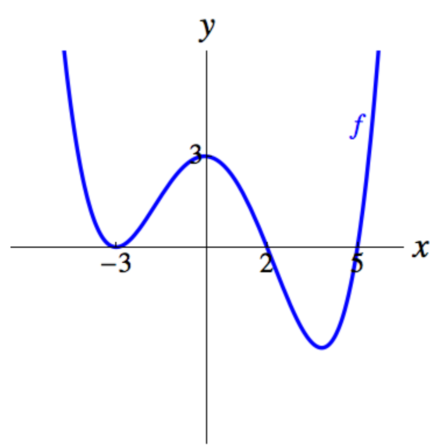
5) 0, 1, 2, 3, or 4

6) If a is any nonzero real number, we can use anything of the form: $ax^2(x+3)(x-5)$, or $ax(x+3)^2(x-5)$, or $ax(x+3)(x-5)^2$.

7) If a is any nonzero real number, we can use anything of the form: $a(x+2)^2(x-4)^3$.

8) f is a polynomial function, so it is continuous on $[1, 2]$. $f(1)=-1$, so $f(1)<0$. $f(2)=1$, so $f(2)>0$. Therefore, by the Intermediate Value Theorem, f has a zero between 1 and 2.

9)
   a)



   b) $f(-4)>0$, $f(0)>0$, $f(3)<0$, and $f(6)>0$

SECTION 2.3: LONG AND SYNTHETIC POLYNOMIAL DIVISION

1) $3x-5+\dfrac{4x-1}{2x^2+4x+3}$

2) $4x+7-\dfrac{9x+2}{3x^2-2}$

3) $-x^2+5+\dfrac{3x^2-x+2}{7x-x^3}$, which some may rewrite as $5-x^2+\dfrac{3x^2-x+2}{7x-x^3}$.
Hint: Reorder the terms in the numerator and in the denominator.

4) $x^3+\dfrac{x}{x^2+x+1}$. (Note: $x^3+\dfrac{x^2-x}{x^3-1}$ simplifies to this.)

5)
a) $3x^2-2x+8+\dfrac{11}{x-4}$
b) $f(x)=(x-4)\left(3x^2-2x+8\right)+11$
c) 11
d) $f(4)\;=\;(4-4)\left(3(4)^2-2(4)+8\right)+11\;=\;0+11\;=\;11$
e) $f(4)\;=\;3(4)^3-14(4)^2+16(4)-21\;=\;11$

6) $5x^4-4x+3-\dfrac{7}{x+2}$. Hint: There is a "missing term" in the numerator.

7) Perform the division $\dfrac{x^3+1}{x+1}$ by letting $k=-1$. We obtain: $\dfrac{x^3+1}{x+1}=x^2-x+1$.
Therefore, $x^3+1=(x+1)\left(x^2-x+1\right)$.

8) x^3+x^2+x+1

9) $x^{n-1}+x^{n-2}+\ldots+x+1$

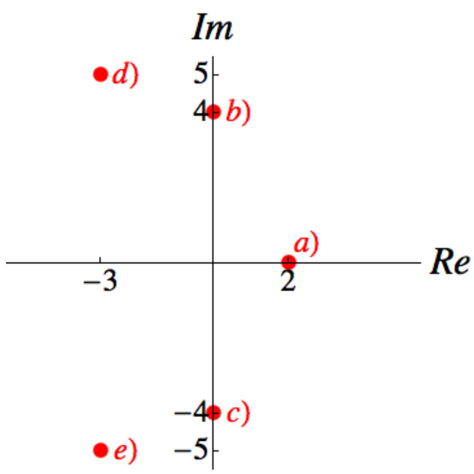
10) a) $3x^2+x-1+\dfrac{5}{2x+1}$; b) same as in a). Hint: Reorder the terms of $f(x)$.

11) a) $15x^2-7x-2$; b) same as in a); c) 0; d) 0; e) $(x-3)(5x+1)(3x-2)$;
f) 3, $-\dfrac{1}{5}$, and $\dfrac{2}{3}$. In increasing order: $-\dfrac{1}{5}$, $\dfrac{2}{3}$, and 3.

SECTION 2.4: COMPLEX NUMBERS

1) a) $i\sqrt{7}$; b) $3i$; c) $4i\sqrt{3}$

2)



3) 0

4) a) 1; b) i; c) $-i$; d) -1; e) 1

5) a) $6i\sqrt{2}$; b) -20 (No, not equivalent); c) $-9-40i$

6) a) $6-5i$; b) $2+i\sqrt{7}$; c) -7; d) πi

7) They are reflections of each other about the real axis.

8) a) $\frac{3}{5}-\frac{2}{5}i$; b) $\frac{13}{25}-\frac{34}{25}i$; c) $-\frac{4}{9}+\frac{7}{9}i\sqrt{5}$

9) $-4i\sqrt{2}$ and $4i\sqrt{2}$

10) $\frac{2-i\sqrt{11}}{3}$ and $\frac{2+i\sqrt{11}}{3}$

SECTION 2.5: FINDING ZEROS OF POLYNOMIAL FUNCTIONS

1)

a) $\pm 1,\ \pm 2,\ \pm\frac{1}{3},\ \pm\frac{2}{3}$

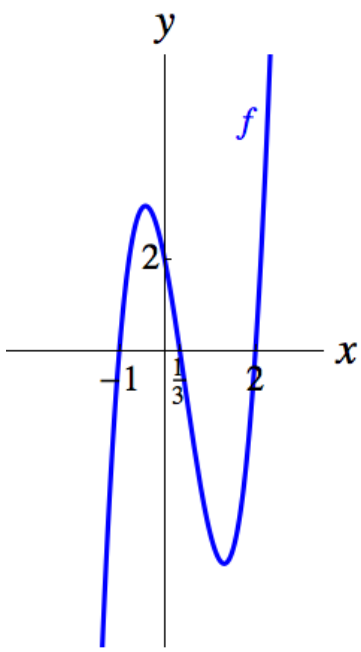
b) The remainder is -28, so $f(-2) = -28$, not 0.

c) The remainder is 0, so $f(-1) = 0$.

d) $(x+1)(3x-1)(x-2)$

e) -1, $\frac{1}{3}$, and 2.

f)



2)

a) $\pm 1,\ \pm 2,\ \pm 3,\ \pm 4,\ \pm 6,\ \pm 9,\ \pm 12,\ \pm 18,\ \pm 36,\ \pm\frac{1}{2},\ \pm\frac{3}{2},\ \pm\frac{9}{2}$

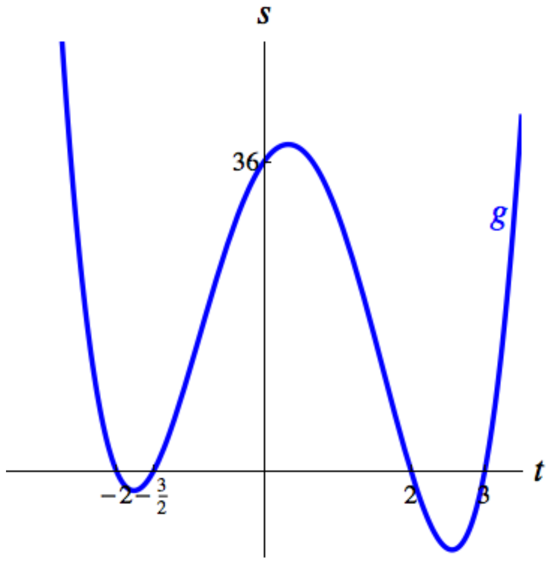
b) The remainder is 132, so $g(4) = 132$. No, 4 is not a zero.

c) The remainder is 0, so $g(3) = 0$. Yes, 3 is a zero.

d) $(t-3)(2t+3)(t+2)(t-2)$

e) 3, $-\frac{3}{2}$, -2, and 2. In increasing order: -2, $-\frac{3}{2}$, 2, and 3.

f)



(Note: The axes are scaled differently.)

g) $\pm 1,\ \pm 2,\ \pm 3,\ \pm 4,\ \pm 6,\ \pm 12,\ \pm\frac{1}{2},\ \pm\frac{3}{2}$. Eliminated: $\pm 9,\ \pm 18,\ \pm 36,\ \pm\frac{9}{2}$.

h) The remainder is 0, so $-\frac{3}{2}$ is a zero of c and of g. We obtain the same factorization as in d); observe that $2\left(t+\frac{3}{2}\right)=2t+3$. The factorization $g(t)=2(t-3)\left(t+\frac{3}{2}\right)(t+2)(t-2)$ is a factorization over $\mathbb{Q}$, and it is in Linear Factorization Theorem (LFT) Form; see Part F.

3) $\left(x^2+\pi\right)\left(x+\sqrt{\pi}\right)\left(x-\sqrt{\pi}\right)$

4) $\left(x+\sqrt[3]{5}\right)\left[x^2-\left(\sqrt[3]{5}\right)x+\sqrt[3]{25}\right]$

5) $\left(x+i\sqrt{7}\right)\left(x-i\sqrt{7}\right)$

6)

a) $4x^2\left(x^2+4\right)\left(x^2-3\right)$. The only rational zero is 0 (with multiplicity 2).

b) $4x^2\left(x^2+4\right)\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)$. The real zeros are 0 (w/mult. 2), $-\sqrt{3}$, and $\sqrt{3}$.

c) $4x^2(x+2i)(x-2i)\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)$. The complex zeros are 0 (w/mult. 2), $-2i$, $2i$, $-\sqrt{3}$, and $\sqrt{3}$.

7) $-7i$ must be a zero. x^2+49 must be a factor; it is prime (irreducible) over $\mathbb{R}$.

8) $4+5i$ must be a zero. $x^2-8x+41$ must be a factor; it is prime (irreducible) over $\mathbb{R}$.

9) The zeros are 0 (with multiplicity 3) and 2 (with multiplicity 2).

10) The zeros are $3i$ (with multiplicity 2) and $-3i$ (with multiplicity 2).

11) 0, 2, 4, or 6. Remember that any imaginary zeros of such polynomials form complex-conjugate pairs. **Distinct** real zeros: 0, 1, 2, 3, 4, 5, or 6.

12) 1, 3, 5, 7, or 9. Remember that any imaginary zeros of such polynomials form complex-conjugate pairs. **Distinct** real zeros: 1, 2, 3, 4, 5, 6, 7, 8, or 9.

13)

a) $\pm 1,\ \pm 3,\ \pm 7,\ \pm 21$

b) $(x-3)(x^2+2x+7)$

c) $3,\ -1-i\sqrt{6}$, and $-1+i\sqrt{6}$

d) $(x-3)\left(x-\left(-1-i\sqrt{6}\right)\right)\left(x-\left(-1+i\sqrt{6}\right)\right)$, or $(x-3)\left(x+1+i\sqrt{6}\right)\left(x+1-i\sqrt{6}\right)$

14)

a) $\pm 1,\ \pm 2,\ \pm 4,\ \pm 8,\ \pm\frac{1}{2},\ \pm\frac{1}{4}$

b) $(r+2)^2\left(4r^2-3r+2\right)$

c) -2 (with multiplicity 2), $\frac{3-i\sqrt{23}}{8}$, and $\frac{3+i\sqrt{23}}{8}$

d) $4(r+2)^2\left(r-\frac{3-i\sqrt{23}}{8}\right)\left(r-\frac{3+i\sqrt{23}}{8}\right)$

15) a) 2 or 0; b) 2; c) 1; d) 1

16) a) 4; b) 3 or 1 (it turns out there is one); c) 2 or 0 (it turns out there are none)

SECTION 2.6: RATIONAL FUNCTIONS

1) a) NONE; b) $x=-\frac{3}{2}$ and $x=1$; c) $y=0$; d) $\frac{1}{5}$, or $\left(\frac{1}{5},0\right)$; e) $\frac{1}{3}$, or $\left(0,\frac{1}{3}\right)$

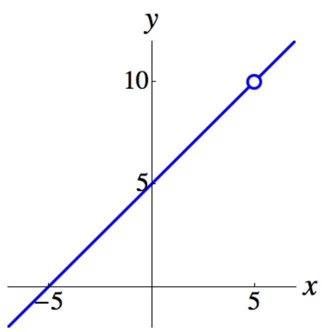
2) a) $f(x)=\frac{2(x-2)}{x+3}$, $(x\neq 0 \text{ and } x\neq 2)$

b) $\left(0,-\frac{4}{3}\right)$ and $(2,0)$; c) $x=-3$; d) $y=2$; e) NONE; f) NONE

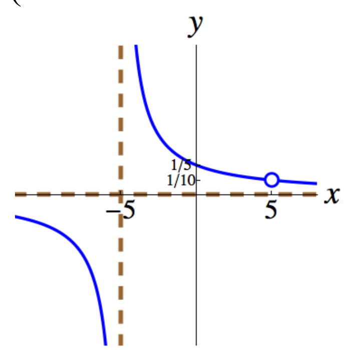
3) $y=3x-5$

4) 6

5)



6) (Axes are scaled differently.)



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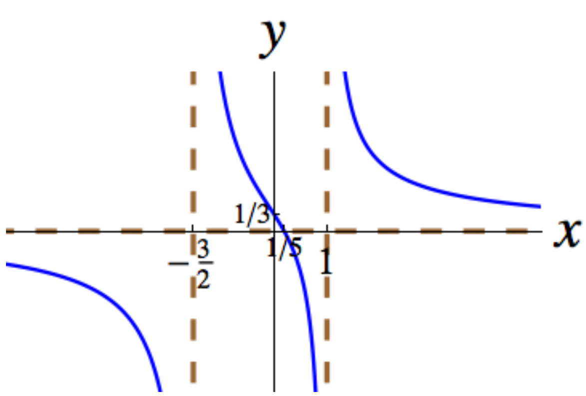

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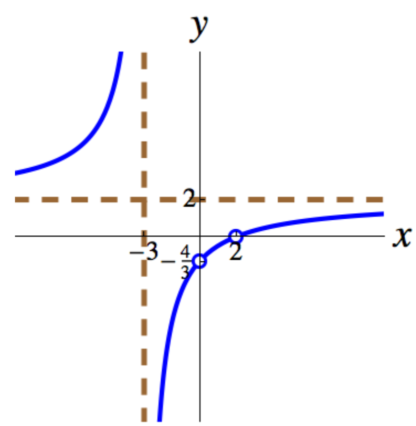


SECTION 2.7: NONLINEAR INEQUALITIES

1) $(-\infty,-3)\cup(1,\infty)$

2) $[-4,4]$

3) $(-\infty,-5)\cup(5,\infty)$

4) $(-\infty,\infty)$

5) $\left[-\frac{1}{3},2\right]$

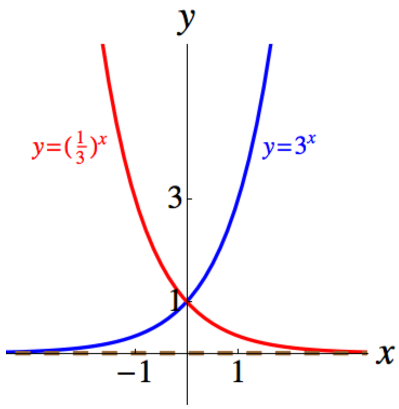
6) $\text{Dom}(f)=\left\{x\in\mathbb{R}\,\middle|\,x=0 \text{ or } x\geq 1\right\}$. Note: $\text{Dom}(f)=\{0\}\cup[1,\infty)$.

CHAPTER 3:

Exponential and Logarithmic Functions

SECTION 3.1: EXPONENTIAL FUNCTIONS AND THEIR GRAPHS

1)



2) $\text{Dom}(f) = (-\infty, \infty)$. $\text{Range}(f) = (0, \infty)$

3) 2.718282

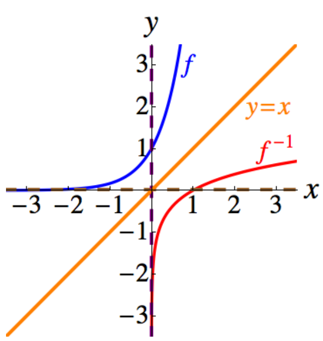
4) \$2362.60

5) a) \$2377.48; b) \$2378.88

SECTION 3.2: LOGARITHMIC FUNCTIONS AND THEIR GRAPHS

1) a) 4; b) $\frac{1}{3}$; c) -2; d) $-\frac{1}{2}$; e) 3; f) 9; g) 0; h) 1; i) 12; j) p

2) $f^{-1}(x) = \log_6 x$



3) $\text{Dom}(g) = (0, \infty)$. $\text{Range}(g) = (-\infty, \infty)$

4) $\text{Dom}(h) = (-5, \infty)$. $\text{Range}(h) = (-\infty, \infty)$

5) $(-\infty, -8) \cup (8, \infty)$

SECTION 3.3: MORE PROPERTIES OF LOGARITHMS

1) 2
2)
   a) $4\ln x + 3\ln y$
   b) $3\log_2(p+2) - 5$
   c) $6 + \frac{1}{3}\ln x - 7\ln y$
   d) $4 + 5\log a - 3\log b - 4\log c$
   e) $9\log_7 x + (\log_7 x)^3 + 10\log_7 y$
3)
   a) $\log_3(x^2 y^4)$
   b) $\log\left[\frac{a^3}{(b-c)^4}\right]$
   c) $\ln\left(\frac{x^3 y^4}{z^{3/4} w^5}\right)$, or $\ln\left(\frac{x^3 y^4}{\sqrt[4]{z^3} \cdot w^5}\right)$
4) It must lie between 4 and 5.
5) 4.8227

SECTIONS 3.4 AND 3.5: EXPONENTIAL AND LOGARITHMIC EQUATIONS AND MODELS

1)
   a) $\{\log_3 12\}$, or $\left\{\frac{\log 12}{\log 3}\right\}$, or $\left\{\frac{\ln 12}{\ln 3}\right\}$, approximately $\{2.2619\}$
   b) $\{3\}$
   c) $\left\{-\frac{2}{7}\right\}$
   d) $\{\ln 4\}$, approximately $\{1.3863\}$
   e) $\left\{\ln\left(\frac{3}{2}\right), \ln 5\right\}$, approximately $\{0.40547, 1.6094\}$

2) $\dfrac{\ln\left(\dfrac{30}{17}\right)}{0.042} = \dfrac{1000\ln\left(\dfrac{30}{17}\right)}{42} = \dfrac{500\ln\left(\dfrac{30}{17}\right)}{21}$ years old, or $\dfrac{500(\ln 30 - \ln 17)}{21}$ years old, which is approximately 13.5 years old.

3) $\dfrac{\ln 2}{0.042} = \dfrac{1000\ln 2}{42} = \dfrac{500\ln 2}{21}$ years old, which is approximately 16.5 years old. This is about three years longer than the time from Exercise 2. This makes sense, because it takes longer for the fund to grow to \$3400 than it does to grow to \$3000.

4) $\dfrac{\ln 3}{0.042} = \dfrac{1000\ln 3}{42} = \dfrac{500\ln 3}{21}$ years old, which is approximately 26.2 years old.

5)

a) $\dfrac{\ln\left(\dfrac{1}{32}\right)}{-0.472} = -\dfrac{1000\ln\left(\dfrac{1}{32}\right)}{472} = -\dfrac{125(\ln 1 - \ln 32)}{59} = \dfrac{125\ln 32}{59}$ hours, which is approximately 7.343 hours.

b) About 7:21pm

6)

a) $\{125\}$

b) $\{e^7 - 2\}$, approximately $\{1094.6\}$

c) $\{7\}$. Additional Problem: If we remove the assumption, we obtain $\left\{-\dfrac{13}{2},\ 7\right\}$.

(Absolute value symbols should be used when applying the Power Rule of Logarithms, because the exponent is even.)

d) $\{1, 4\}$

e) $\{6\}$

f) $\{3\}$

g) $\varnothing$. Note: $-\dfrac{625}{624}$ is an "extraneous solution" that must be rejected.

h) $\left\{\dfrac{-1+\sqrt{1+4e}}{2}\right\}$, approximately $\{1.2229\}$