(Answers for Chapter 2: Polynomial and Rational Functions) A.2.1

CHAPTER 2:

Polynomial and Rational Functions

SECTION 2.1: QUADRATIC FUNCTIONS
(AND PARABOLAS)

1)

a) Upward;  b) \((1, -9)\);  c) \(x = 1\)

d) \(-8\), or \((0, -8)\)

e) \(-2\) and \(4\), or \((-2, 0)\) and \((4, 0)\)

f) \((2, -8)\)

g)

[Graph of a quadratic function with key points labeled]

2)

a) Downward;  b) \(\left(-\frac{11}{4}, \frac{243}{8}\right)\), or \((-2.75, 30.375)\);  c) \(x = -\frac{11}{4}\), or \(x = -2.75\)

d) \(-15\), or \((0, -15)\)

e) \(-5\) and \(-\frac{1}{2}\), or \((-5, 0)\) and \(\left(-\frac{1}{2}, 0\right)\)
3)  
   a) Upward;  b) \((2, 3)\);  c) \(x = 2\)  
   d) 7, or \((0, 7)\)  
   e) None  

4) a) \(y = (x + 4)^2 - 6\);  b) Upward;  c) \((-4, -6)\)  

5) a) \(y = -4(x - 3)^2 - 1\);  b) Downward;  c) \((3, -1)\)  

6) a) \(y = 2\left(x - \frac{7}{2}\right)^2 + \frac{5}{2}\), or \(y = 2\left(x - 3.5\right)^2 + 2.5\)  
   b) Upward  
   c) \(\left(\frac{7}{2}, \frac{5}{2}\right)\), or \((3.5, 2.5)\)  

7) \(y = -\frac{2}{9}(x - 6)^2 - 3\), or \(y = -\frac{2}{9}x^2 + \frac{8}{3}x - 11\)  

8)  
   a) \(-$2000\) (i.e., a net loss of $2000)  
   b) 7 widgets  
   c) $450  
   d) 4 widgets and 10 widgets  

9)  
   a) 384 feet  
   b) \(\frac{5}{2}\) seconds, or 2.5 seconds  
   c) 484 feet  
   d) 8 seconds
SECTION 2.2: POLYNOMIAL FUNCTIONS OF HIGHER DEGREE

1) 
   a) $\infty, -\infty$
   b) $\infty, \infty$
   c) $-\infty, -\infty$
   d) $-\infty, \infty$

2) 3 or 1

3) 6, 4, 2, or 0

4) The degree of $f$ is odd and is at least 5. The leading coefficient of $f(x)$ is positive. $f(0) = 1$. $f$ has exactly one distinct negative real zero, and it has odd multiplicity.

5) 0, 1, 2, 3, or 4

6) If $a$ is any nonzero real number, we can use anything of the form: $ax^2(x + 3)(x - 5)$, or $ax(x + 3)^2(x - 5)$, or $ax(x + 3)(x - 5)^2$.

7) If $a$ is any nonzero real number, we can use anything of the form: $a(x + 2)^2(x - 4)^3$.

8) $f$ is a polynomial function, so it is continuous on $[1, 2]$. $f(1) = -1$, so $f(1) < 0$. $f(2) = 1$, so $f(2) > 0$. Therefore, by the Intermediate Value Theorem, $f$ has a zero between 1 and 2.

9) 
   a) 
   b) $f(-4) > 0$, $f(0) > 0$, $f(3) < 0$, and $f(6) > 0$
SECTION 2.3: LONG AND SYNTHETIC POLYNOMIAL DIVISION

1) \(3x - 5 + \frac{4x - 1}{2x^2 + 4x + 3}\)

2) \(4x + 7 - \frac{9x + 2}{3x^2 - 2}\)

3) \(-x^2 + 5 + \frac{3x^2 - x + 2}{7x - x^3}\), which some may rewrite as \(5 - x^2 + \frac{3x^2 - x + 2}{7x - x^3}\).
   Hint: Reorder the terms in the numerator and in the denominator.

4) \(x^3 + \frac{x}{x^2 + x + 1}\). (Note: \(x^3 + \frac{x^2 - x}{x^3 - 1}\) simplifies to this.)

5) 
   a) \(3x^2 - 2x + 8 + \frac{11}{x - 4}\)
   b) \(f(x) = (x - 4)(3x^2 - 2x + 8) + 11\)
   c) 11
   d) \(f(4) = (4 - 4)(3(4)^2 - 2(4) + 8) + 11 = 0 + 11 = 11\)
   e) \(f(4) = 3(4)^3 - 14(4)^2 + 16(4) - 21 = 11\)

6) \(5x^4 - 4x + 3 - \frac{7}{x + 2}\). Hint: There is a “missing term” in the numerator.

7) Perform the division \(\frac{x^3 + 1}{x + 1}\) by letting \(k = -1\). We obtain: \(\frac{x^3 + 1}{x + 1} = x^2 - x + 1\).
   Therefore, \(x^3 + 1 = (x + 1)(x^2 - x + 1)\).

8) \(x^3 + x^2 + x + 1\)

9) \(x^{n-1} + x^{n-2} + \ldots + x + 1\)

10) a) \(3x^2 + x - 1 + \frac{5}{2x + 1}\); b) same as in a). Hint: Reorder the terms of \(f(x)\).

11) a) \(15x^2 - 7x - 2\); b) same as in a); c) 0; d) 0; e) \((x - 3)(5x + 1)(3x - 2)\)
    f) 3, \(-\frac{1}{5}\), and \(\frac{2}{3}\). In increasing order: \(-\frac{1}{5}\), \(\frac{2}{3}\), and 3.
SECTION 2.4: COMPLEX NUMBERS

1) a) \( i\sqrt{7} \); b) \( 3i \); c) \( 4i\sqrt{3} \)

2)

3) 0

4) a) 1; b) \( i \); c) \( -i \); d) \( -1 \); e) 1

5) a) \( 6i\sqrt{2} \); b) \(-20\) (No, not equivalent); c) \(-9 - 40i\)

6) a) \( 6 - 5i \); b) \( 2 + i\sqrt{7} \); c) \(-7 \); d) \(\pi i\)

7) They are reflections of each other about the real axis.

8) a) \( \frac{3}{5} - \frac{2}{5}i \); b) \( \frac{13}{25} - \frac{34}{25}i \); c) \( -\frac{4}{9} + \frac{7}{9}i\sqrt{5} \)

9) \(-4i\sqrt{2}\) and \(4i\sqrt{2}\)

10) \(\frac{2 - i\sqrt{11}}{3}\) and \(\frac{2 + i\sqrt{11}}{3}\)
SECTION 2.5: FINDING ZEROS OF POLYNOMIAL FUNCTIONS

1)

a) \( \pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3} \)

b) The remainder is \(-28\), so \( f(-2) = -28 \), not 0.

c) The remainder is 0, so \( f(-1) = 0 \).

d) \((x+1)(3x-1)(x-2)\)

e) \(-1, \frac{1}{3}, \) and 2.

f)

\[
\text{Graph of } f(x) \text{ with roots at } -1, \frac{1}{3}, \text{ and } 2.
\]

2)

a) \( \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2} \)

b) The remainder is 132, so \( g(4) = 132 \). No, 4 is not a zero.

c) The remainder is 0, so \( g(3) = 0 \). Yes, 3 is a zero.

d) \((t-3)(2t+3)(t+2)(t-2)\)

e) \(3, \ -\frac{3}{2}, \ -2, \) and 2. In increasing order: \(-2, \ -\frac{3}{2}, 2, \) and 3.
g) \( \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2} \). Eliminated: \( \pm 9, \pm 18, \pm 36, \pm \frac{9}{2} \).

h) The remainder is 0, so \(-\frac{3}{2}\) is a zero of \(c\) and of \(g\). We obtain the same factorization as in d); observe that \(2\left(t + \frac{3}{2}\right) = 2t + 3\). The factorization
\[
g(t) = 2(t - 3)\left(t + \frac{3}{2}\right)(t + 2)(t - 2)
\]
is a factorization over \(\mathbb{Q}\), and it is in Linear Factorization Theorem (LFT) Form; see Part F.

3) \( (x^2 + \pi)(x + \sqrt{\pi})(x - \sqrt{\pi}) \)
4) \( (x + \sqrt{5})\left[x^2 - (\sqrt{5})x + \sqrt{25}\right] \)
5) \( (x + i\sqrt{7})(x - i\sqrt{7}) \)

6)
   a) \( 4x^2(x^2 + 4)(x^2 - 3) \). The only rational zero is 0 (with multiplicity 2).
   b) \( 4x^2(x^2 + 4)(x + \sqrt{3})(x - \sqrt{3}) \). The real zeros are 0 (w/mult. 2), \(-\sqrt{3}\), and \(\sqrt{3}\).
   c) \( 4x^2(x + 2i)(x - 2i)(x + \sqrt{3})(x - \sqrt{3}) \). The complex zeros are 0 (w/mult. 2), \(-2i\), \(2i\), \(-\sqrt{3}\), and \(\sqrt{3}\).
7) \(-7i\) must be a zero. \(x^2 + 49\) must be a factor; it is prime (irreducible) over \(\mathbb{R}\).

8) \(4 + 5i\) must be a zero. \(x^2 - 8x + 41\) must be a factor; it is prime (irreducible) over \(\mathbb{R}\).

9) The zeros are 0 (with multiplicity 3) and 2 (with multiplicity 2).

10) The zeros are \(3i\) (with multiplicity 2) and \(-3i\) (with multiplicity 2).

11) 0, 2, 4, or 6. Remember that any imaginary zeros of such polynomials form complex-conjugate pairs. **Distinct** real zeros: 0, 1, 2, 3, 4, 5, or 6.

12) 1, 3, 5, 7, or 9. Remember that any imaginary zeros of such polynomials form complex-conjugate pairs. **Distinct** real zeros: 1, 2, 3, 4, 5, 6, 7, 8, or 9.

13) 
   a) \(\pm 1, \pm 3, \pm 7, \pm 21\)
   b) \((x-3)(x^2+2x+7)\)
   c) 3, \(-1-i\sqrt{6}\), and \(-1+i\sqrt{6}\)
   d) \((x-3)(x-(-1-i\sqrt{6}))(x-(-1+i\sqrt{6}))\), or \((x-3)(x+1+i\sqrt{6})(x+1-i\sqrt{6})\)

14) 
   a) \(\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}, \pm \frac{1}{4}\)
   b) \((r+2)^2\left(4r^2-3r+2\right)\)
   c) \(-2\) (with multiplicity 2), \(\frac{3-i\sqrt{23}}{8}\), and \(\frac{3+i\sqrt{23}}{8}\)
   d) \(4(r+2)^2\left(r-\frac{3-i\sqrt{23}}{8}\right)\left(r-\frac{3+i\sqrt{23}}{8}\right)\)

15) a) 2 or 0; b) 2; c) 1; d) 1

16) a) 4; b) 3 or 1 (it turns out there is one); c) 2 or 0 (it turns out there are none)
SECTION 2.6: RATIONAL FUNCTIONS

1) a) NONE; b) \( x = -\frac{3}{2} \) and \( x = 1 \); c) \( y = 0 \); d) \( \frac{1}{5} \), or \( \left( \frac{1}{5}, 0 \right) \); e) \( \frac{1}{3} \), or \( \left( 0, \frac{1}{3} \right) \)

2) a) \( f(x) = \frac{2(x-2)}{x+3}, \ (x \neq 0 \text{ and } x \neq 2) \)

   b) \( \left( 0, -\frac{4}{3} \right) \) and \( (2, 0) \); c) \( x = -3 \); d) \( y = 2 \); e) NONE; f) NONE

3) \( y = 3x - 5 \)

4) 6

5) \( \begin{align*}
\end{align*} \)

6) (Axes are scaled differently.)

SECTION 2.7: NONLINEAR INEQUALITIES

1) \((-\infty, -3) \cup (1, \infty)\)

2) \([-4, 4]\)

3) \((-\infty, -5) \cup (5, \infty)\)

4) \((-\infty, \infty)\)

5) \([-\frac{1}{3}, 2]\)

6) \(\text{Dom}(f) = \{ x \in \mathbb{R} \mid x = 0 \text{ or } x \geq 1 \}\). Note: \(\text{Dom}(f) = \{0\} \cup [1, \infty)\).
CHAPTER 3:

Exponential and Logarithmic Functions

SECTION 3.1: EXPONENTIAL FUNCTIONS AND THEIR GRAPHS

1) 

2) \( \text{Dom}(f) = (\infty, \infty) \). \( \text{Range}(f) = (0, \infty) \)

3) 2.718282

4) $2362.60

5) a) $2377.48; b) $2378.88

SECTION 3.2: LOGARITHMIC FUNCTIONS AND THEIR GRAPHS

1) a) 4; b) \( \frac{1}{3} \); c) \(-2\); d) \(-\frac{1}{2}\); e) 3; f) 9; g) 0; h) 1; i) 12; j) \( p \)

2) \( f^{-1}(x) = \log_6 x \)

3) \( \text{Dom}(g) = (0, \infty) \). \( \text{Range}(g) = (-\infty, \infty) \)

4) \( \text{Dom}(h) = (-5, \infty) \). \( \text{Range}(h) = (-\infty, \infty) \)

5) \((-\infty, -8) \cup (8, \infty)\)
SECTION 3.3: MORE PROPERTIES OF LOGARITHMS

1) 2
2) 
   a) $4 \ln x + 3 \ln y$
   b) $3 \log_2 (p + 2) - 5$
   c) $6 + \frac{1}{3} \ln x - 7 \ln y$
   d) $4 + 5 \log a - 3 \log b - 4 \log c$
   e) $9 \log_7 x + (\log_7 x)^3 + 10 \log_7 y$
3) 
   a) $\log_3 (x^2 y^4)$
   b) $\log \left( \frac{a^3}{(b - c)^4} \right)$
   c) $\ln \left( \frac{x^3 y^4}{z^{3/4} w^5} \right)$, or $\ln \left( \frac{x^3 y^4}{4 \sqrt[3]{z} \cdot w^5} \right)$
4) It must lie between 4 and 5.
5) 4.8227

SECTIONS 3.4 AND 3.5:
EXPONENTIAL AND LOGARITHMIC EQUATIONS AND MODELS

1) 
   a) $\{ \log_3 12 \}$, or $\left\{ \frac{\log 12}{\log 3} \right\}$, or $\left\{ \frac{\ln 12}{\ln 3} \right\}$, approximately $\{ 2.2619 \}$
   b) $\{ 3 \}$
   c) $\left\{ -\frac{2}{7} \right\}$
   d) $\{ \ln 4 \}$, approximately $\{ 1.3863 \}$
   e) $\left\{ \ln \left( \frac{3}{2} \right), \ln 5 \right\}$, approximately $\{ 0.40547, 1.6094 \}$
(Answers for Chapter 3: Exponential and Logarithmic Functions) A.3.3.

2) \[
\frac{\ln \left( \frac{30}{17} \right)}{0.042} = \frac{1000 \ln \left( \frac{30}{17} \right)}{42} = \frac{500 \ln \left( \frac{30}{17} \right)}{21} \text{ years old, or } \frac{500(\ln 30 - \ln 17)}{21} \text{ years old, which is approximately 13.5 years old.}
\]

3) \[
\frac{\ln 2}{0.042} = \frac{1000 \ln 2}{42} = \frac{500 \ln 2}{21} \text{ years old, which is approximately 16.5 years old.}
\]
This is about three years longer than the time from Exercise 2. This makes sense, because it takes longer for the fund to grow to $3400 than it does to grow to $3000.

4) \[
\frac{\ln 3}{0.042} = \frac{1000 \ln 3}{42} = \frac{500 \ln 3}{21} \text{ years old, which is approximately 26.2 years old.}
\]

5)

a) \[
\frac{\ln \left( \frac{1}{32} \right)}{-0.472} = -\frac{1000 \ln \left( \frac{1}{32} \right)}{472} = -\frac{125(\ln 1 - \ln 32)}{59} = \frac{125 \ln 32}{59} \text{ hours, which is approximately 7.343 hours.}
\]

b) About 7:21pm

6)

a) \{125\}

b) \{e^7 - 2\}, approximately \{1094.6\}

c) \{7\}. Additional Problem: If we remove the assumption, we obtain \(-\frac{13}{2}, \ 7\).

(Absolute value symbols should be used when applying the Power Rule of Logarithms, because the exponent is even.)

d) \{1, 4\}

e) \{6\}

f) \{3\}

g) \emptyset. Note: \(-\frac{625}{624}\) is an “extraneous solution” that must be rejected.

h) \left\{-1 + \sqrt{1 + 4e} \right\}, approximately \{1.2229\}