

CHAPTER 4:

Introduction to Trigonometry

SECTION 4.1: ANGLES

1) a) $48^\circ 15'$, b) $341^\circ 42'07''$

2) a) 22.5° , b) 102.298°

3) a) $\frac{4\pi}{9}$, b) $-\frac{\pi}{5}$, c) 4.735

4) a) 18° , b) -135° , c) 227.1°

5) 4.5 inches

6) 4 meters

7) $\frac{1}{3}$ [of a radian]

8)

$\frac{\pi}{6}$	QI	Acute	$\frac{7\pi}{6}$	QIII	None of these
$\frac{\pi}{3}$	QI	Acute	$\frac{3\pi}{2}$	Quadrantal	None of these
$\frac{\pi}{2}$	Quadrantal	Right	$\frac{5\pi}{3}$	QIV	None of these
$\frac{2\pi}{3}$	QII	Obtuse	$\frac{7\pi}{4}$	QIV	None of these
$\frac{3\pi}{4}$	QII	Obtuse			
π	Quadrantal	None of these			

9) a) 65° , b) $\frac{\pi}{3}$

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10) a) 130° , b) $\frac{3\pi}{4}$

11)

a) For example, $-\frac{11\pi}{6}$, $\frac{13\pi}{6}$, and $\frac{25\pi}{6}$; also, $\frac{37\pi}{6}$.

In general: $\frac{\pi}{6} + 2\pi n$ ($n \in \mathbb{Z}$).

b) For example, -315° , 405° , and 765° ; also, 1125° .

In general: $45^\circ + 360n^\circ$ ($n \in \mathbb{Z}$).

SECTIONS 4.2-4.4: TRIGONOMETRIC FUNCTIONS (VALUES AND IDENTITIES)

1) a) $\frac{5}{3}$, b) $\frac{5}{4}$, c) $\frac{3}{4}$, d) $\frac{4}{3}$, e) 1

2) a) $\frac{2\sqrt{29}}{29}$, b) $\frac{5\sqrt{29}}{29}$, c) $\frac{2}{5}$, d) $\frac{\sqrt{29}}{2}$, e) $\frac{\sqrt{29}}{5}$, f) $\frac{5}{2}$, g) 1

3) a) $\frac{12}{13}$, b) $\frac{5}{12}$, c) $\frac{13}{5}$, d) $\frac{13}{12}$, e) $\frac{12}{5}$, f) 1

4) a) 3 m, b) $3\sqrt{2}$ m

5) a) $\frac{4\sqrt{3}}{3}$ m, b) $\frac{8\sqrt{3}}{3}$ m

6) $\cos(70^\circ)$

7)

θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$	$\csc(\theta)$	$\sec(\theta)$	$\cot(\theta)$
0	0	1	0	und.	1	und.
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{2}$	1	0	und.	1	und.	0

- 8) a) $\sin(80^\circ)$, b) $\cos(70^\circ)$, c) $\tan(80^\circ)$, d) $\csc(70^\circ)$,
e) $\csc(10^\circ)$ (Hint: This equals $\sec(80^\circ)$ by the Cofunction Identities.)

- 9) a) QII, b) QIII, c) QIV, d) QIII

10) a) $\frac{\pi}{6}$; b) QIV; c) $\sin(\theta) = -\frac{1}{2}$, $\cos(\theta) = \frac{\sqrt{3}}{2}$, $\tan(\theta) = -\frac{\sqrt{3}}{3}$, $\csc(\theta) = -2$,

$$\sec(\theta) = \frac{2\sqrt{3}}{3}, \cot(\theta) = -\sqrt{3}$$

11) a) 45° ; b) QII; c) $\sin(\theta) = \frac{\sqrt{2}}{2}$, $\cos(\theta) = -\frac{\sqrt{2}}{2}$, $\tan(\theta) = -1$, $\csc(\theta) = \sqrt{2}$,
 $\sec(\theta) = -\sqrt{2}$, $\cot(\theta) = -1$

12) a) $\frac{4\pi}{3}$; b) $\frac{\pi}{3}$; c) QIII; d) $\sin(\theta) = -\frac{\sqrt{3}}{2}$, $\cos(\theta) = -\frac{1}{2}$, $\tan(\theta) = \sqrt{3}$,
 $\csc(\theta) = -\frac{2\sqrt{3}}{3}$, $\sec(\theta) = -2$, $\cot(\theta) = \frac{\sqrt{3}}{3}$

13) a) 150° ; b) 30° ; c) QII; d) $\sin(\theta) = \frac{1}{2}$, $\cos(\theta) = -\frac{\sqrt{3}}{2}$, $\tan(\theta) = -\frac{\sqrt{3}}{3}$,
 $\csc(\theta) = 2$, $\sec(\theta) = -\frac{2\sqrt{3}}{3}$, $\cot(\theta) = -\sqrt{3}$

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14)

θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$	$\csc(\theta)$	$\sec(\theta)$	$\cot(\theta)$
$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	-2	$-\frac{\sqrt{3}}{3}$
π	0	-1	0	und.	-1	und.
$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	$-\sqrt{2}$	$-\sqrt{2}$	1
$\frac{3\pi}{2}$	-1	0	und.	-1	und.	0
$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	-2	$\frac{2\sqrt{3}}{3}$	$-\sqrt{3}$
$\frac{5\pi}{2}$	1	0	und.	1	und.	0
$\frac{11\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	2	$-\frac{\sqrt{3}}{3}$
9π	0	-1	0	und.	-1	und.

15) a) $\sqrt{3}$, b) und., c) 0, d) $-\sqrt{2}$

16) i.

17) ii.

18) ii.

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19)

Left Side	Right Side	Type of ID
$\csc(\theta)$	$\frac{1}{\sin(\theta)}$	Reciprocal ID
$\sec(\theta)$	$\frac{1}{\cos(\theta)}$	Reciprocal ID
$\cot(\theta)$	$\frac{1}{\tan(\theta)}$	Reciprocal ID
$\tan(\theta)$	$\frac{\sin(\theta)}{\cos(\theta)}$	Quotient ID
$\cot(\theta)$	$\frac{\cos(\theta)}{\sin(\theta)}$	Quotient ID
$\sin\left(\frac{\pi}{2} - \theta\right)$	$\cos(\theta)$	Cofunction ID
$\cot\left(\frac{\pi}{2} - \theta\right)$	$\tan(\theta)$	Cofunction ID
$\sin(-\theta)$	$-\sin(\theta)$	Even / Odd (Negative-Angle) ID
$\cos(-\theta)$	$\cos(\theta)$	Even / Odd (Negative-Angle) ID
$\tan(-\theta)$	$-\tan(\theta)$	Even / Odd (Negative-Angle) ID
$\sin^2(\theta) + \cos^2(\theta)$	1	Pythagorean ID
$\tan^2(\theta) + 1$	$\sec^2(\theta)$	Pythagorean ID
$1 + \cot^2(\theta)$	$\csc^2(\theta)$	Pythagorean ID

20) a) -0.3 , b) 0.3 , c) $\frac{\sqrt{91}}{10} \approx 0.954$

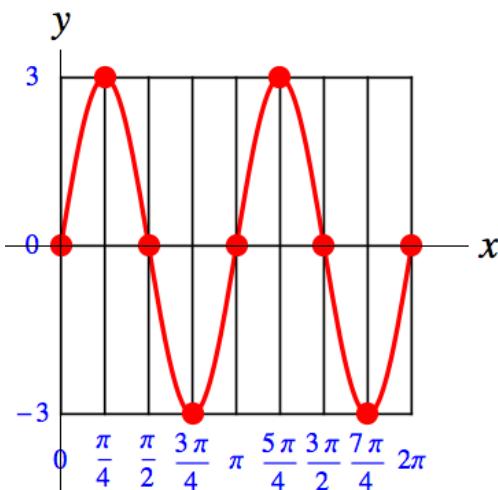
21) $\frac{7}{5}$

22) $\frac{9}{4}$

SECTION 4.5: GRAPHS OF SINE AND COSINE FUNCTIONS

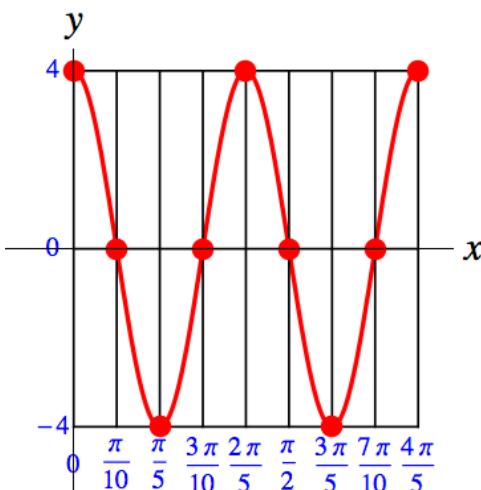
The coordinate axes may be scaled differently.

1) a)



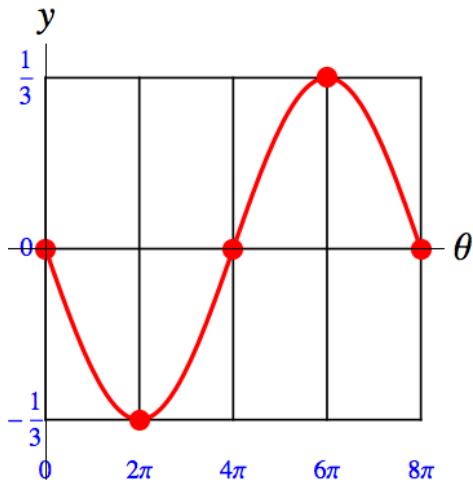
b) 3; c) π ; d) \mathbb{R} , or $(-\infty, \infty)$; e) $[-3, 3]$

2) a)



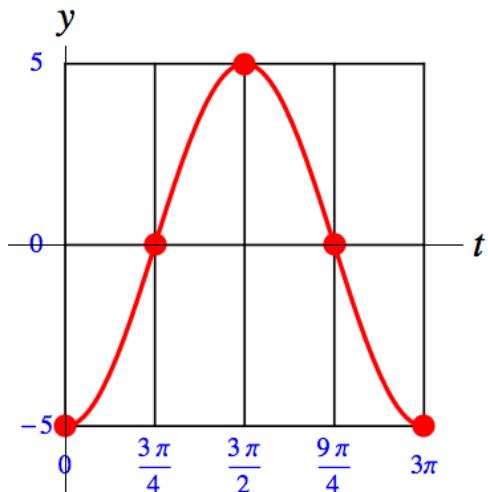
b) 4; c) $\frac{2\pi}{5}$; d) \mathbb{R} , or $(-\infty, \infty)$; e) $[-4, 4]$

3)



- b) $\frac{1}{3}$; c) 8π ; d) \mathbb{R} , or $(-\infty, \infty)$; e) $\left[-\frac{1}{3}, \frac{1}{3}\right]$

4)

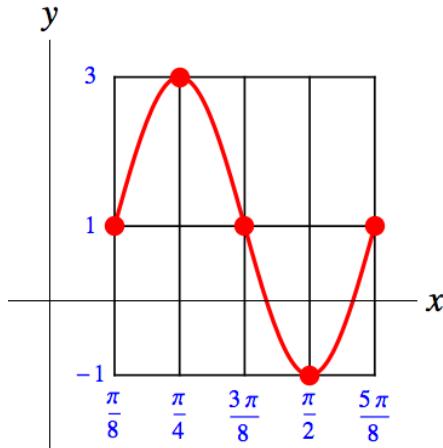


- b) 5; c) 3π ; d) \mathbb{R} , or $(-\infty, \infty)$; e) $[-5, 5]$

5) $a = -2$, $b = \frac{5}{2}$

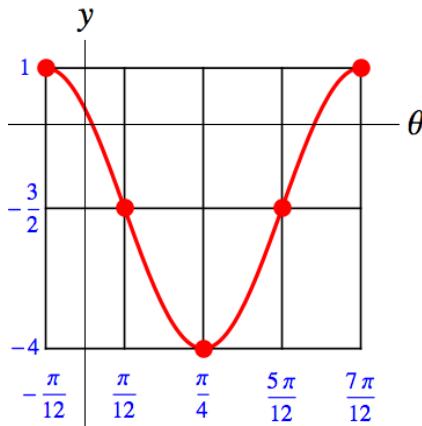
6) $a = \frac{2}{3}$, $b = 4$

7) a)



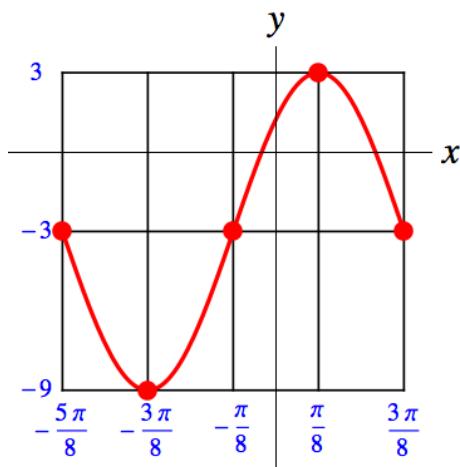
b) 2; c) $\frac{\pi}{2}$; d) $\frac{\pi}{8}$; e) \mathbb{R} , or $(-\infty, \infty)$; f) $[-1, 3]$

8)



b) $\frac{5}{2}$; c) $\frac{2\pi}{3}$; d) $-\frac{\pi}{12}$; e) \mathbb{R} , or $(-\infty, \infty)$; f) $[-4, 1]$

9)

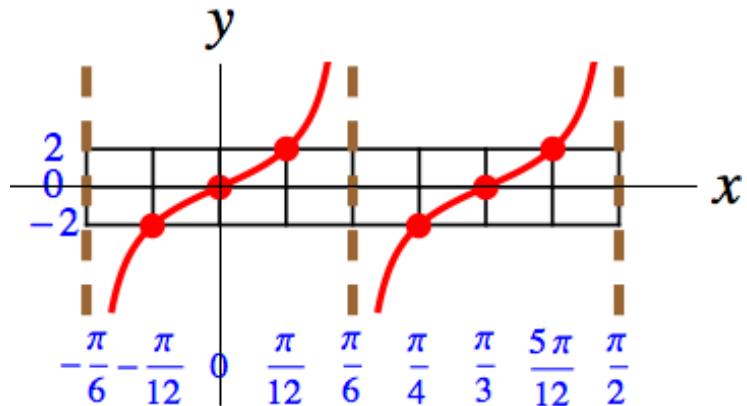


b) 6; c) π ; d) $\frac{3\pi}{8}$, which may be preferable to $-\frac{5\pi}{8}$, since $\frac{3\pi}{8}$ has a lesser absolute value and is closer to 0; e) \mathbb{R} , or $(-\infty, \infty)$; f) $[-9, 3]$

SECTION 4.6: GRAPHS OF OTHER TRIGONOMETRIC FUNCTIONS

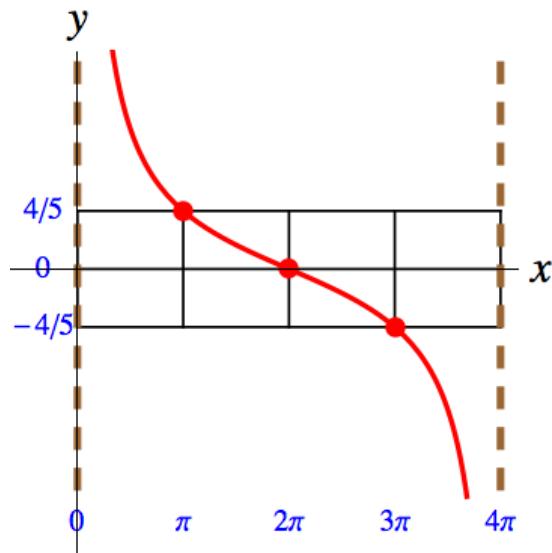
The coordinate axes may be scaled differently.

1) a)



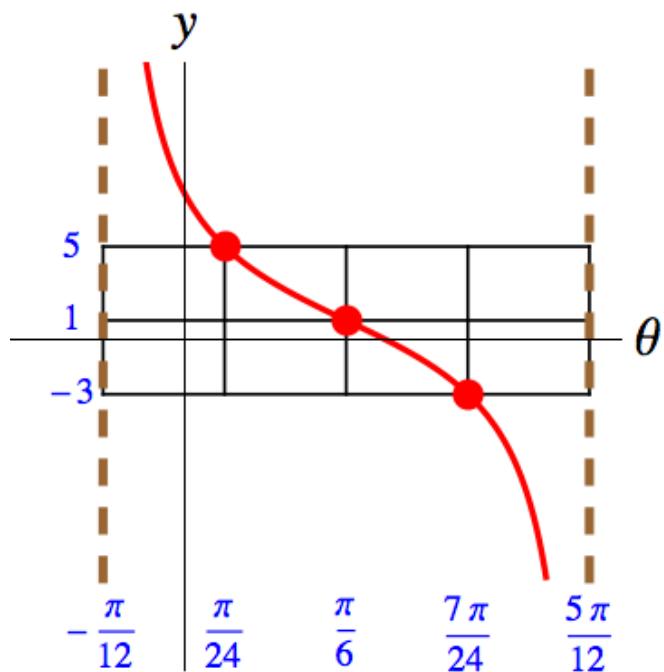
b) $\frac{\pi}{3}$; c) \mathbb{R} , or $(-\infty, \infty)$

2) a)



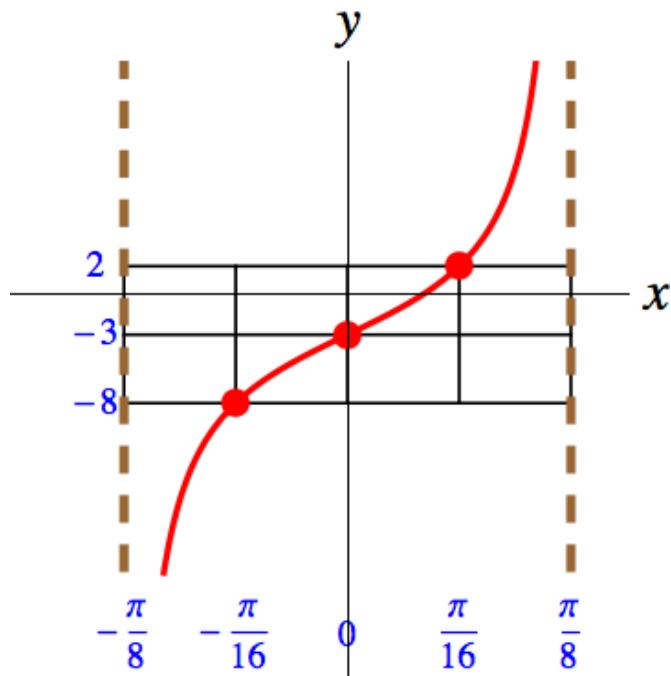
b) 4π

3) a)



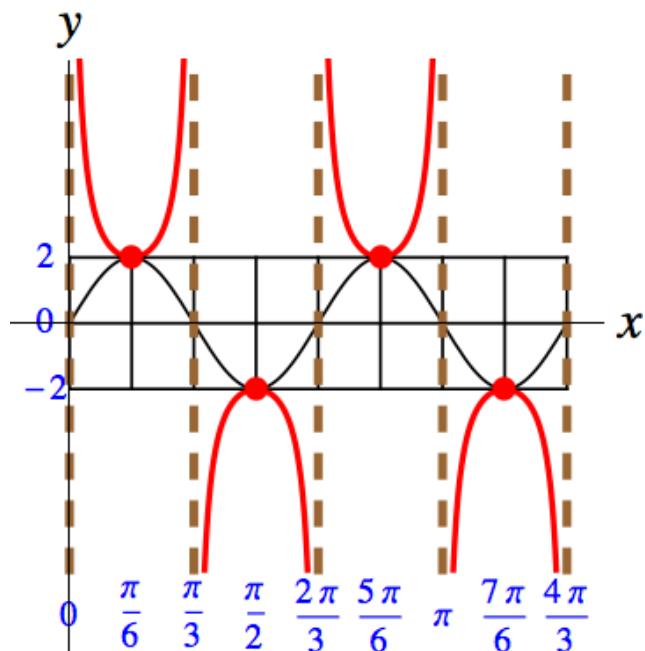
b) $\frac{\pi}{2}$; c) $\frac{\pi}{6}$

4) a)



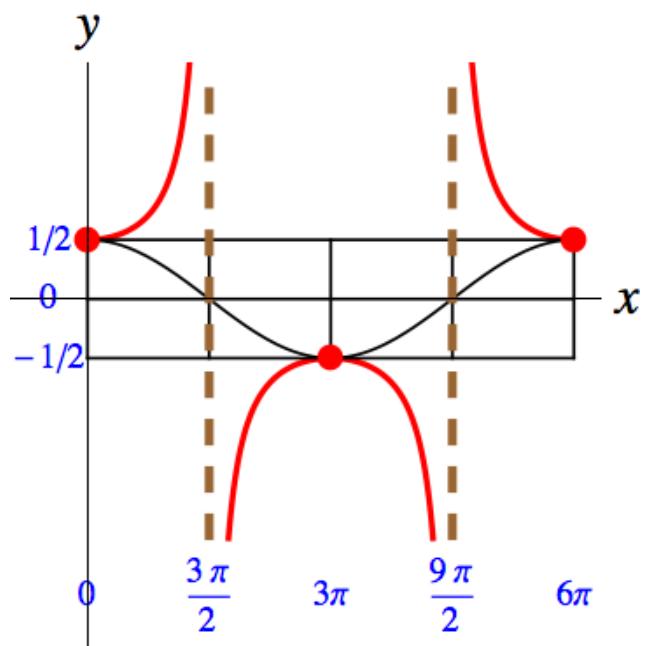
b) $\frac{\pi}{4}$; c) $-\frac{\pi}{8}$

5) a)



b) $\frac{2\pi}{3}$

6) a)



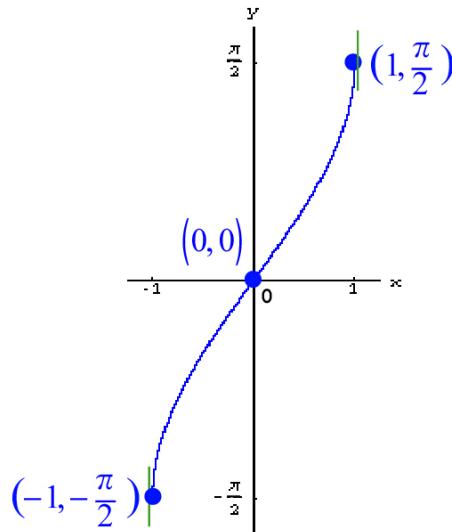
b) 6π

7)

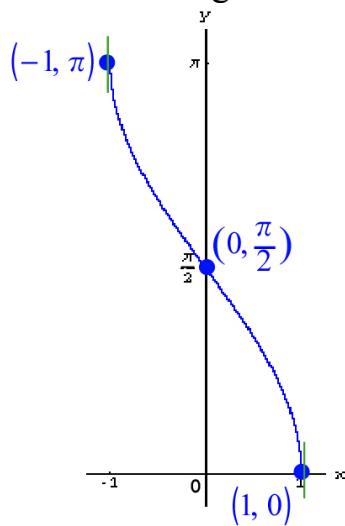
$f(x)$	Domain	Range
$\sin(x)$	$(-\infty, \infty)$	$[-1, 1]$
$\cos(x)$	$(-\infty, \infty)$	$[-1, 1]$
$\tan(x)$	Use set-builder form. $\left\{ x \in \mathbb{R} \mid x \neq \frac{\pi}{2} + \pi n \ (n \in \mathbb{Z}) \right\}$	$(-\infty, \infty)$
$\csc(x)$	Use set-builder form. $\left\{ x \in \mathbb{R} \mid x \neq \pi n \ (n \in \mathbb{Z}) \right\}$	$(-\infty, -1] \cup [1, \infty)$
$\sec(x)$	Use set-builder form. $\left\{ x \in \mathbb{R} \mid x \neq \frac{\pi}{2} + \pi n \ (n \in \mathbb{Z}) \right\}$	$(-\infty, -1] \cup [1, \infty)$
$\cot(x)$	Use set-builder form. $\left\{ x \in \mathbb{R} \mid x \neq \pi n \ (n \in \mathbb{Z}) \right\}$	$(-\infty, \infty)$

SECTION 4.7: INVERSE TRIGONOMETRIC FUNCTIONS

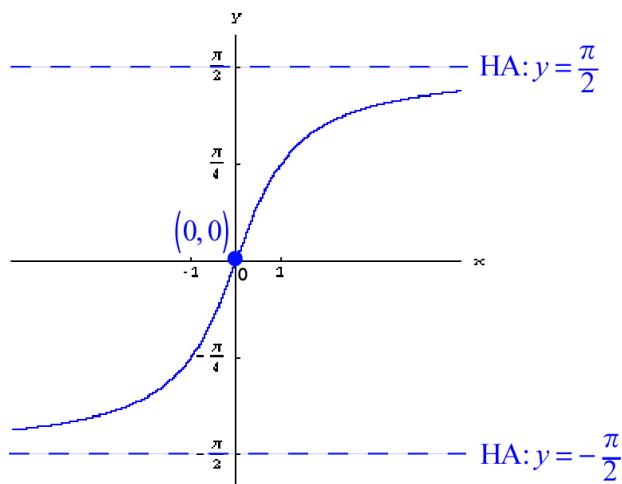
- 1) (The vertical tangent lines are indicated at the endpoints.)



- 2) (The vertical tangent lines are indicated at the endpoints.)



- 3)



4)

$f(x)$	Domain	Range
$\sin^{-1}(x)$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}(x)$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}(x)$	$(-\infty, \infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

5)

a) $\frac{\pi}{6}$

b) $-\frac{\pi}{3}$

c) undefined

d) $\frac{3\pi}{4}$

e) $-\frac{\pi}{4}$

f) $\frac{2}{7}$

g) undefined

h) -2

i) $\frac{\pi}{5}$

j) $\frac{5\pi}{6}$

k) $-\frac{\pi}{3}$

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6)

a) $\frac{x}{\sqrt{25-x^2}}$, or $\frac{x\sqrt{25-x^2}}{25-x^2}$

b) $\frac{1}{\sqrt{x^2+1}}$, or $\frac{\sqrt{x^2+1}}{x^2+1}$

7)

- a) 17.46° and 162.5° ; 0.3047 and 2.837
- b) 197.5° and 342.5° ; 3.446 and 5.978
- c) 72.54° and 287.5° ; 1.266 and 5.017
- d) 107.5° and 252.5° ; 1.875 and 4.408
- e) 84.29° and 264.3° ; 1.471 and 4.613
- f) 95.71° and 275.7° ; 1.670 and 4.812

SECTION 4.8: APPLICATIONS

1) $a \approx 2.22$ in, $c \approx 3.53$ in

2) 3.91 ft

3) a) 7.11 m, b) 25.8°

4) a) 1930 ft, b) 61.9° , c) 80.5°