CHAPTER 7:

Systems and Inequalities

SECTIONS 7.1-7.3: SYSTEMS OF EQUATIONS

1)
   a) 1

   b) \( \{(-1, 6)\} \). Hint: If the first equation is solved for \( y \) in terms of \( x \), we eventually obtain 
   \[ 5x - 3(5 - x) = -23. \]

   c) \( \{(-1, 6)\} \). Hint: If we multiply both sides of the first equation by 3, we obtain
   \[
   \begin{align*}
   3x + 3y &= 15 \\
   5x - 3y &= -23
   \end{align*}
   \]

2)
   a) \( \{(-1, 1)\} \)

   b)

   ![Graph of a circle and a line](image)

3)
   a) a circle and a parabola [that opens upward], respectively

   b) 2

   c) \( \left\{ \left(1, \frac{\sqrt{2}}{2}\right), \left(-1, \frac{\sqrt{2}}{2}\right) \right\} \).

   Warning: ‘±’ notation could be considered ambiguous. Checks may be necessary to eliminate extraneous solutions.
4)  
   a) a parabola [that opens to the right] and a parabola [that opens to the left], respectively.
   
   b) 2
   
   c) \[\{(2, \sqrt{2}), (2, -\sqrt{2})\}\].
   
   Warning: ‘±’ notation could be considered ambiguous.

5)  
   a) Hint: Rewrite \[x^2 + y = 0\] as \[y = -x^2\]. Rewrite \[y - x^2 = 1\] as \[y = x^2 + 1\].

   b) \(\emptyset\). (Observe that the two graphs in a) do not intersect.)

   c) \(\emptyset\). You may obtain the equation \[x^2 = -\frac{1}{2}\], which has no real solutions for \(x\).

6)  
   a) \[\{(-4,52), (2,10)\}\]
   
   b) \[\left\{\left(-\frac{7}{5}, -\frac{1}{10}\right), \left(-1, \frac{1}{2}\right)\right\}\]
   
   c) \(\emptyset\). You may obtain the equation \[x^2 = \frac{5}{2}\], but then there are no corresponding real values for \(y\).

7) Additional Problem: \(\emptyset\)
SECTION 7.4: PARTIAL FRACTIONS

1) 
   a) \[ \frac{A}{x+4} + \frac{B}{x-3} + \frac{Cx+D}{x^2+1} \]
   b) \[ \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-1} + \frac{E}{(x-1)^2} + \frac{Fx+G}{x^2+3} + \frac{Hx+I}{(x^2+3)^2} \]
   c) \[ \frac{A}{t} + \frac{B}{t^2} + \frac{C}{2t+5} + \frac{D}{(2t+5)^2} + \frac{E}{(2t+5)^3} + \frac{Ft+G}{2t^2+5} + \frac{Ht+I}{t^2+t+1} \]

2) 
   a) \[ \frac{4}{x-3} - \frac{1}{x-2} \]
   b) \[ \frac{3}{x+5} - \frac{2}{x+1} + \frac{1}{x-2} \]
   c) \[ \frac{2}{x} + \frac{6}{x^2} + \frac{5}{2x+1} \]
   d) \[ \frac{1}{x-4} + \frac{5}{(x-4)^2} \]
   e) \[ \frac{6}{x+2} + \frac{2x+3}{x^2+1} \]
   f) \[ \frac{4}{x-5} + \frac{x}{x^2+3} \]
   g) \[ -\frac{3}{x} - \frac{-2x-5}{x^2+x+1}, \text{ or, equivalently, } -\frac{3}{x} - \frac{2x+5}{x^2+x+1} \]
   h) \[ \frac{5t-1}{t^2+4} - \frac{4}{(t^2+4)^2} \]

3) No, because the rational expression on the left-hand side is improper. (The student should multiply out the denominator and then perform Long Division; the idea of using repeated Synthetic Division can be confusing.)
CHAPTER 8:  
Matrices and Determinants

SECTION 8.1: MATRICES and SYSTEMS OF EQUATIONS

1) a) \(3 \times 2\), 6 entries; b) \(4 \times 5\), 20 entries; c) \(3 \times 3\) (order 3 square), 9 entries

2) a) 
\[
\begin{bmatrix}
3 & -1 & 18 \\
1 & 2 & -1 \\
\end{bmatrix}
\]

b) \(2 \times 2\) (order 2 square)

c) \(2 \times 1\)

d) 
\[
\begin{bmatrix}
1 & 2 & -1 \\
3 & -1 & 18 \\
\end{bmatrix}
\]

e) 
\[
\begin{bmatrix}
1 & 2 & -1 \\
0 & -7 & 21 \\
\end{bmatrix}
\]

f) 
\[
\begin{bmatrix}
1 & 2 & -1 \\
0 & 1 & -3 \\
\end{bmatrix}
\]

g) \[
\begin{align*}
x + 2y &= -1 \\
y &= -3
\end{align*}
\]

h) \(\{(5, -3)\}\)

i) 
\[
\begin{align*}
3(5) - (-3) &= 18 \\
(5) + 2(-3) &= -1
\end{align*}
\]

3) \(\left\{\left(\frac{1}{2}, -\frac{5}{2}\right)\right\}\)

4) \(\{-2, -7\}\)
5) Additional Problem: \( \left\{ \left( \frac{2}{3}, -\frac{4}{3} \right) \right\} \)

6) \( \emptyset \)

7) The system has infinitely many solutions.
   Note 1: The solutions correspond to the points on the line \( x + 3y = 6 \) in the \( xy \)-plane.
   Note 2: The solution set can be given by: \( \{(x, y) \in \mathbb{R}^2 \mid x + 3y = 6\} \).

8) \( \{(2, 4, -5)\} \)

9) \( \{(7, -2, 1)\} \)

10) \( \left\{ \left( -\frac{1}{2}, 0, \frac{3}{2} \right) \right\} \)

11) \( \{(1, -1, -3)\} \)

12) \( \emptyset \)

13) a) Yes; b) Yes

14) a) Yes; b) No

15) a) No; b) No

16) a) No; b) No

17) Additional Problem: a) \( \{(3, -2)\} \); b) \( \{(4, 8, 1)\} \); c) \( \{(-1, 5, 3)\} \)
SECTION 8.2: OPERATIONS WITH MATRICES

1) a) \[
\begin{bmatrix}
6 & 3 \\
10 & 1 \\
-2 & \pi + \sqrt{5}
\end{bmatrix}
\]; b) \[
\begin{bmatrix}
-31 & 9 \\
-26 & 24 \\
57 & 3\pi - 4\sqrt{5}
\end{bmatrix}
\]

2) \[22\], or simply the scalar 22 (depending on context)

3) a) \[
\begin{bmatrix}
7 & -2 \\
-7 & 6
\end{bmatrix}
\]; b) \[
\begin{bmatrix}
4 & -1 \\
-8 & 9
\end{bmatrix}
\]; c) No

4) a) \[
\begin{bmatrix}
16 & 5 & 8 \\
-15 & 9 & 0
\end{bmatrix}
\]; b) Undefined; c) \[
\begin{bmatrix}
-3 & 20 & 10 \\
-6 & -4 & -5 \\
4 & 9 & 7
\end{bmatrix}
\]

5) a) \(8 \times 7\)
b) Multiply the fifth row of \(A\) and the sixth column of \(B\), in that order.

6) \(n = p\) and \(m = q\)

7) \[
\begin{bmatrix}
4 & 0 & 0 \\
0 & 9 & 0 \\
0 & 0 & 16
\end{bmatrix}
\]
a) \[
\begin{bmatrix}
1024 & 0 & 0 \\
0 & 59,049 & 0 \\
0 & 0 & 1,048,576
\end{bmatrix}
\]

8) a) \(3 \times 4\); b) Undefined; c) Undefined; d) \(3 \times 7\); e) \(3 \times 7\); f) \(3 \times 7\); in fact, the expressions in e) and f) are equivalent.

9) \[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

10) Additional Problem:

a) \(A = \begin{bmatrix} 4 & 4 & -3 \\ 5 & 7 & -13 \\ 1 & 2 & -5 \end{bmatrix}\); b) \(X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\); c) \(B = \begin{bmatrix} 7 \\ -9 \\ -6 \end{bmatrix}\);

d) The coefficient matrix was \(I_3\). The RHS was \[
\begin{bmatrix}
-1 \\
5 \\
3
\end{bmatrix}
\], which is the solution for \(X\).
SECTION 8.3: THE INVERSE OF A SQUARE MATRIX

Additional Problems:

1) $I_2$, which is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2)
   a) $\begin{bmatrix} \frac{2}{7} & \frac{1}{7} \\ \frac{1}{7} & \frac{1}{7} \end{bmatrix}$
   b) $A$ is noninvertible

3) $\{ (4, -3) \}$. Hint: Use $X = A^{-1} B$.

4) $\{ (1, 0, -1) \}$. Hint: Use $X = A^{-1} B$.

5) a) 7; b) $\begin{bmatrix} \frac{2}{7} & \frac{1}{7} \\ \frac{1}{7} & \frac{3}{7} \end{bmatrix}$, just as in Exercise 2a

6) a) 0; b) $A^{-1}$ is undefined, just as in Exercise 2b

7) Hint: Show that $(AB)(B^{-1} A^{-1}) = I_n$, or $(B^{-1} A^{-1})(AB) = I_n$.

8) Hint: Show that $AA^{-1} = I_2$. 
SECTION 8.4: THE DETERMINANT OF A SQUARE MATRIX

1)
   a) $-4$
   b) $2$
   c) $-2$. $C$ is obtained by switching the two rows of $B$, and $\det(C) = -\det(B)$.
   d) $20$. $D$ is obtained by multiplying the first row of $B$ by 10, and $\det(C) = 10 \det(B)$.
   e) $2$. $E$ is obtained by taking the transpose of $B$, and $\det(E) = \det(B^T) = \det(B)$.
   f) $-26$
   g) $0$
   h) $0$
   i) $0$. (This is exemplified by g) and h), in which the second row is a multiple of the first row. For both, $a = 4$ and $b = 5$. In g), $c = 0$. In h), $c = 10$.)
   j) $e^{2x} - xe^x$, or $e^x(e^x - x)$
   k) $1$

2)
   a) $92$. Hint: $+(-12) + (60) + (-3) - (-45) - (6) - (-8) = 92$.
   b) $92$. Hint: $+2(-9) - (4)(-17) + 3(14) = 92$.
   c) $92$. Hint: $-(4)(-17) + (-3)(-11) - (1)(9) = 92$.

3)
   a) $-17$. Hint: $+12 + 0 + 24 - 8 - 0 - 45 = -17$.
   b) $-17$. Hint: $-3(7) + 4(1) + 0 = -17$.
   c) $-17$. Hint: $-2(-8) - 0 + 3(-11) = -17$.

4) $0$. If one row of a square matrix is a multiple of another row of that matrix, then the determinant of the matrix is 0.

5) $6000$. The determinant of an upper (or lower) triangular matrix is equal to the product of the entries along the main diagonal.

6) $-66$. Hint: Expand by cofactors along the third column. $-(2)(33) = -66$.

7) $0$. Hint: Expand by cofactors along the fourth row.

8) $\{2, 3\}$. That is, the eigenvalues are 2 and 3.
SECTION 8.5: APPLICATIONS OF DETERMINANTS

Additional Problems:

1) \[ \begin{bmatrix} \frac{1}{2}, -\frac{5}{2} \end{bmatrix} \]

2) \[ \{(4, -3)\} \]

3) 
   a) 20 square meters (or m\(^2\))
   b) 29 square meters (or m\(^2\))
   c) 0. This implies that the vectors are parallel (i.e., the position vectors are collinear).

4) 12 square meters (or m\(^2\))
CHAPTER 9:

Discrete Mathematics

SECTION 9.1: SEQUENCES AND SERIES, and
SECTION 9.6: COUNTING PRINCIPLES

1) \( a_1 = 2, \ a_2 = 6, \ a_3 = 12 \)
2) \( a_1 = -2, \ a_2 = 4, \ a_3 = -6, \ a_4 = 8 \)
3) \( a_1 = 1, \ a_2 = -3, \ a_3 = 5, \ a_4 = -7 \)
4) 720
5) 120 ways
6) 21 ways
7) 252 ways
8)
   \[
   \begin{align*}
   &a) \ (n+1)!
   
b) \ (n+2)(n+1), \ or \ n^2 + 3n + 2 \\
   &c) \ \frac{1}{n(n+1)}, \ or \ \frac{1}{n^2 + n} \\
   &d) \ \frac{1}{(3n+3)(3n+2)(3n+1)(3n)(3n-1)}
   \end{align*}
   \]
9) \( a_1 = 4, \ a_2 = 14, \ a_3 = 24, \ a_4 = 34 \)
10) \( a_1 = 2, \ a_2 = 1, \ a_3 = \frac{1}{2}, \ a_4 = \frac{1}{4} \)
11) \( a_1 = -1, \ a_2 = -5, \ a_3 = -17, \ a_4 = -53 \)
12) \( a_1 = 2, \ a_2 = 3, \ a_3 = 6, \ a_4 = 18, \ a_5 = 108 \)
13) 10
14) 140
15) \( \frac{5}{12} \)
16) 120
17) 
   a) \( a_n = n + 6 \)
   b) \( a_n = 5n \)
   c) \( a_n = 3n + 1 \)
   d) \( a_n = \frac{1}{n!} \)
   e) \( a_n = \frac{7}{n^2} \)
   f) \( a_n = (-1)^{n-1} \frac{2n}{2n+1}, \) or \( a_n = (-1)^{n+1} \frac{2n}{2n+1} \)
   g) \( a_n = (-1)^n 2^n, \) or \( a_n = (-2)^n \)

18) 
   a) \( \sum_{k=1}^{6} 3k \)
   b) \( \sum_{k=1}^{\infty} (-1)^{k-1} \frac{1}{4^k}, \) or \( \sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{4^k}, \) or \( \sum_{k=1}^{\infty} -\left( -\frac{1}{4} \right)^k \)

SECTION 9.2: ARITHMETIC SEQUENCES and PARTIAL SUMS

1) a) \(-5\); b) \(6\)

2) a) \(\frac{3}{2}\); b) \(-\frac{1}{2}\)

3) a) 7, 10, 13, 16; b) 46; c) 184

4) 
   a) \( a_n = 7 - 5n \), which is simplified from \( a_n = 2 + (n - 1)(-5) \)
   b) \(-1928\)

5) 860. (Hint: First find \( d \). \( d = 7 \).)
SECTION 9.3: GEOMETRIC SEQUENCES, PARTIAL SUMS, and SERIES

1) a) 4; b) 5

2) a) $\frac{6}{7}$; b) $-\frac{1}{3}$

3) a) 5, -20, 80, -320; b) -255; c) -20, 971, 520; d) No (What is $r$?)

4)
   a) $a_n = \left(\frac{2}{9}\right)\left(\frac{3}{2}\right)^{n-1}$, or $a_n = \frac{3^{n-3}}{2^{n-2}}$
   b) $\frac{2187}{256}$
   c) $a_6 = \frac{27}{16}$, $a_7 = \frac{81}{32}$, $a_8 = \frac{243}{64}$, $a_9 = \frac{729}{128}$, $a_{10} = \frac{2187}{256}$
   d) No (What is $r$?)

5)
   a) $a_2 = -\frac{6}{5}$, $a_3 = \frac{12}{25}$. (Hint: First find $r$. $r = -\frac{2}{5}$.)
   b) Yes; 0 (What is $r$?)

6) The series diverges. (What is $r$?)

7) The series diverges. (What is $r$?)

8) The series converges. (What is $r$?) The sum is $\frac{15}{7}$.

9) $\left\{ x \in \mathbb{R} \mid -\frac{1}{3} < x < \frac{1}{3} \right\}$, or the interval $\left( -\frac{1}{3}, \frac{1}{3} \right)$

10) a) $\sum_{n=1}^{\infty} 2\left(3^{n-2}\right)$, which is simplified from $\sum_{n=1}^{\infty} \frac{2}{3}\left(3^{n-1}\right)$. It can be rewritten as $\sum_{i=1}^{\infty} 2\left(3^i\right)$.
   (The index of summation could be something other than $n$ or $i$.)
   b) Divergent
   c) No sum
Answers for Chapter 9: Discrete Mathematics

A.9.4.

11) a) \[ \sum_{n=1}^{\infty} 3(-1)^n, \] which is simplified from \[ \sum_{n=1}^{\infty} -3(-1)^{n-1}. \]
   (The index of summation could be something other than \( n \) or \( i \).)

b) Divergent

c) No sum

12) a) \[ \sum_{n=1}^{\infty} 5 \left( -\frac{1}{4} \right)^{n-1}, \] which can be rewritten as \[ \sum_{i=0}^{\infty} 5 \left( -\frac{1}{4} \right)^{i}. \]
   (The index of summation could be something other than \( n \) or \( i \).)

b) Convergent

c) 4

13) \[ \frac{13}{33} \]

14) Additional Problem: \[ \frac{5167}{9990}. \] Hint: Rewrite 0.5172 as 0.5 + 0.0172.

SECTION 9.4: MATHEMATICAL INDUCTION

1) See the notes.

2) 500,500

SECTION 9.5: THE BINOMIAL THEOREM

1) \[ x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5 \]

2) \[ a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6 \]

3) \[ 8x^3 + 36x^2y + 54xy^2 + 27y^3 \]

4) \[ a^4 - 8a^3b + 24a^2b^2 - 32ab^3 + 16b^4 \]

5) \[ 3x^2 + 3xh + h^2. \] Additional Problem: \( f''(x) = 3x^2. \)
CHAPTER 10:  
Conics and Polar Coordinates

SECTION 10.3: ELLIPSES

1)

a) \( \frac{(x - 3)^2}{25} + \frac{(y + 4)^2}{4} = 1 \)

b) \((3, -4)\)

c) \((-2, -4)\) and \((8, -4)\)

d) \((3 - \sqrt{21}, -4)\) and \((3 + \sqrt{21}, -4)\); approximately, \((-1.58, -4)\) and \((7.58, -4)\)

e) \(\frac{\sqrt{21}}{5} \approx 0.917\)

2)

a) \( \frac{(x + 5)^2}{9} + \frac{(y + 1)^2}{16} = 1 \)

b) \((-5, -1)\)

c) \((-5, -5)\) and \((-5, 3)\)

d) \((-5, -1 - \sqrt{7})\) and \((-5, -1 + \sqrt{7})\); approximately, \((-5, -3.65)\) and \((-5, 1.65)\)

e) \(\frac{\sqrt{7}}{4} = 0.661\)
SECTION 10.4: HYPERBOLAS

1) Additional Problem:

2) Additional Problem:
SECTION 10.8: POLAR COORDINATES

1) Cartesian graph paper
   
   ![Cartesian Graph Paper](image1)

   Polar graph paper
   
   ![Polar Graph Paper](image2)

2) Cartesian graph paper
   
   ![Cartesian Graph Paper](image3)

   Polar graph paper
   
   ![Polar Graph Paper](image4)

Additional Problem: \( x^2 - 3x + y^2 = 0 \)