Math 141 Name: _____

MIDTERM 2 – PART 1

(CHAPTERS 2 AND 3: POLYNOMIAL, RATIONAL, EXP'L, LOG FUNCTIONS)
MATH 141 – FALL 2016 – KUNIYUKI
150 POINTS TOTAL: 50 FOR PART 1, AND 100 FOR PART 2

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed.

Unless otherwise specified, give exact answers.
Write units where appropriate in your answers.

PART 1: USING SCIENTIFIC CALCULATORS (50 PTS.)

1) The profit P (in dollars) for the Guzzler car company is given by P or $P(x) = -125x^2 + 5000x - 21{,}875$, where x is the number of cars produced and sold. Assume that the domain of P is $[0, \infty)$, and assume that every car produced is sold. (19 points total)

   a) What is the profit if no cars are produced and sold?
   Hint: The company loses money then.

   b) Use a formula we used in class to find the number of cars (produced and sold) for which profit is maximized.

   c) What is the corresponding maximum profit?

   d) What are the breakeven production levels for the company? That is, how many cars are to be produced and sold if the company's profit is to be \$0? There are two answers; give both.
   Hint: The GCF of $-125x^2 + 5000x - 21{,}875$ is 125 or -125.

2) Consider $f(x) = 3x^3 - 10x^2 + 7x - 12$. Hint: One of the zeros is 3. (17 points)

a) Write the two other complex zeros of f in simplest, standard form. Box in your answers! (13 points)

b) Write the polynomial $f(x)$ as a product of a constant and three linear factors over $\mathbb{C}$, the set of complex numbers. We basically want the Linear Factorization Theorem (LFT) Form of the factorization. (4 points)

3) An exponential growth model for the population of Fredonia is given by: $P(t) = P_0 e^{0.0471t}$, where $P(t)$ is the population t years after January 1, 2010. The population of Fredonia was 64,000 people on January 1, 2010. (10 points total)

a) In how many years after January 1, 2010 will the population of Fredonia be 75,000 people? Give **both** an **exact** answer (which may look ugly; you don't have to simplify it) and an **approximate** answer rounded off to three significant digits. Write units.

b) In what year will the population of Fredonia be 75,000 people? You may use part a).

4) Approximate $\log_5(1684)$ to four decimal places. Use a formula we have discussed in class! (4 points)

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PART 2: NO CALCULATORS ALLOWED! (100 POINTS)

Unless otherwise specified, give exact answers, and graphs are assumed to be in the usual *xy*-plane.

5) The equation of a parabola is given by: $y = -3\left(x-4\right)^2 + 5$. (4 points total)

   a) Which way does the parabola open? Box in one: Upward Downward

   b) What is the vertex of the parabola?

6) Fill in each blank below with ∞ or $-\infty$, as appropriate. (4 points total)

   a) If $f\left(x\right) = 4x^5 - 2x^4 + 3$, then $\lim_{x\to-\infty} f\left(x\right) =$ _____

   b) If $g\left(x\right) = 5x^4 + x^2 - \frac{1}{x}$, then $\lim_{x\to\infty} g\left(x\right) =$ _____

7) Write the list of the possible rational zeros of f, where $f\left(x\right) = 2x^5 + 3x^3 - 6x^2 + 7$, based on the Rational Zero Test (Rational Roots Theorem). You do <u>not</u> have to determine which of these candidates are, in fact, zeros. (6 points)

8) Use Long Division to perform the division: $\frac{6x^5 + 2x^3 - 12x^2 + 4x - 5}{3x^2 + 1}$.

   Write your answer in the form: $\left(\text{polynomial}\right) + \left(\text{proper rational expression}\right)$.

   (11 points)

9) Write a polynomial in x in factored form such that its only zeros are -3 (with multiplicity 2) and 5 (with multiplicity 4). (5 points)

10) Simplify $\frac{1}{3-7i}$ by writing the quotient in standard form. (6 points)

11) If $f(x)$ is a nonzero polynomial with real coefficients such that one of its zeros is $5+7i$, what other complex number must also be a zero of $f(x)$? (2 points)

12) Consider the graph of $y=f(x)$, where $f(x)=\frac{3x^2+2x-1}{9x^2-4}$. If an answer to a part below is none, write "NONE." Box in the answers! (20 points total)

a) Factor the numerator and the denominator of $\frac{3x^2+2x-1}{9x^2-4}$. (4 points)

b) Yes or No: Does the graph of $y=f(x)$ have any holes? (Holes correspond to "removable discontinuities.") Box in one: (2 points)

Yes No

c) Find the equation(s) of the vertical asymptote(s) (VAs), if any. (4 points)

d) Find the equation of the horizontal asymptote (HA), if any. (3 points)

e) Find the x-intercept(s), if any. (4 points)

f) Find the y-intercept, if any. (3 points)

13) Write the domain of f, where $f\left(x\right)=\sqrt{x^2-4x+3}$ using interval form (the form using parentheses and/or brackets). (5 points)

14) Fill in the blank: If $f\left(x\right)=3^x$, then $f^{-1}\left(x\right)=$ _____. (2 points)

15) Simplify the following: (6 points total; 2 points each)

a) $\log_{16}\left(2\right)$

b) $\log\left(1000\right)$

c) $\log_2\left(\frac{1}{8}\right)$

16) Expand and evaluate where appropriate: $\ln\left[\frac{e^4x^5}{\left(y^3\right)\left(\sqrt{z}\right)}\right]$.

Assume $x, y, z>0$. (10 points)

17) Find all real solution(s) of the equation: $\log_2(x) + \log_2(x+2) = 3$.
Write the solution set. Show all work, as in class; do not use trial-and-error!
(12 points)

18) Find the real solution of the equation: $e^{2x} - 11e^x = 0$.
Write the solution set. Show all work! Hint: Factor. (7 points)