Math 141 Name: _____

MIDTERM 2 – PART 1

(CHAPTERS 2 AND 3: POLYNOMIAL, RATIONAL, EXP'L, LOG FUNCTIONS)
MATH 141 – FALL 2018 – KUNIYUKI
150 POINTS TOTAL: 41 FOR PART 1, AND 109 FOR PART 2

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed.

Unless otherwise specified, give exact answers.
Write units where appropriate in your answers.

PART 1: USING SCIENTIFIC CALCULATORS (41 PTS.)

1) The "Vertex Form" for the equation of a parabola in the usual *xy*-plane is given by $y = -3\left(x+4\right)^2 - 6$. (4 points total)

   a) Which way does this parabola open? Box in one:

   Upward Downward

   b) What is the vertex of this parabola?

2) The profit *P* (in dollars) for the Superdoom computer game company is given by *P* or $P\left(x\right) = -20x^2 + 400x - 1500$, where *x* is the number of game DVDs produced and sold. You may assume that the domain of *P* is $\left[0, \infty\right)$.
   For parts a) and b), write units! (8 points total)

   a) Write and use a formula we used in class to find the number of DVDs (produced and sold) for which profit is maximized. (4 points)

   b) What is the corresponding maximum profit? (4 points)

3) Consider $s(r) = r^3 - 3r^2 - 4r + 42$ in parts a) and b) below.

Hint: One of the zeros is -3. (16 points total)

a) Write the two other complex zeros of *s* in simplest, standard form. Show all work, as in class. Box in your answers! (13 points)

b) Write the polynomial $s(r)$ as a product of three linear factors over $\mathbb{C}$, the set of complex numbers. We basically want the Linear Factorization Theorem (LFT) Form of the factorization. (3 points)

4) An exponential decay model for the number of fish remaining in a toxic lake is given by: $P(t) = P_0 e^{-0.0723t}$, where $P(t)$ is the number of fish in the lake t years after January 1, 2000. It was estimated that there were 5800 fish in the lake on January 1, 2000. (9 points total)

   a) In how many years after January 1, 2000 will there be only 1000 fish remaining in the lake? Give **both** an **exact** answer (which may look ugly; you don't have to simplify it) and an **approximate** answer rounded off to three significant digits. Write units! (8 points)

   b) In what year will there be only 1000 fish remaining in the lake? (1 point)

5) Approximate $\log_5(1234)$ to four decimal places. Show work by using a formula we have discussed in class. (4 points)

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Graphs are assumed to be in the usual *xy*-plane.

PART 2: NO CALCULATORS ALLOWED! (109 POINTS)

6) Fill in each blank below with ∞ or $-\infty$. (4 points total; 2 points each)

a) If $f(x) = -2x^3 + 5x^2 - 3 + \frac{1}{x^2}$, then $\lim_{x \to \infty} f(x) =$ _____

b) If $g(x) = 2x^4 + 3x^2 + 2$, then $\lim_{x \to -\infty} g(x) =$ _____

7) How many turning points (TPs) can the graph of $y = f(x)$ have if f is a 4^{th}-degree polynomial function? (3 points)

8) A polynomial (in x) of degree 4 has the following zeros: 0, 3, and -2. Its leading coefficient is 7, and 3 is a zero of multiplicity 2. Write this polynomial; you may leave it in factored form. (5 points)

9) Use Long Division to perform the division: $\frac{8x^4 + 12x^3 - 6x^2 - 7x + 1}{4x^2 - 3}$.
Write your answer in the form: $(\text{polynomial}) + (\text{proper rational expression})$.
(11 points)

10) If $f(x)$ is a nonzero polynomial with real coefficients such that one of its zeros is $3+5i$, what other complex number must also be a zero of $f(x)$? (1 point)

11) Write the list of the possible rational zeros of f, where $f(x)=7x^5+12x^3-4x^2+2$, based on the Rational Zero Test (Rational Roots Theorem). You do not have to determine which of these candidates are, in fact, zeros. (6 points)

12) Factor $x^3-3x^2+9x-27$ completely over $\mathbb{C}$, the set of complex numbers. Hint: You may use Factoring by Grouping first to obtain the shortest solution. (7 points)

13) Simplify i^{447}. (2 points)

14) Consider $f(x)=3x^6-7x^4+2x^3+1$. Using only Descartes's Rule of Signs, … (8 points total)

a) List the possible numbers of **positive** real zeros of f (accounting for multiplicity: double roots are counted twice, for example). (3 points)

b) List the possible numbers of **negative** real zeros of f (accounting for multiplicity: double roots are counted twice, for example). Show work, as in class. (5 points)

15) Consider the graph of $y = \dfrac{(x+1)^2(x+2)}{(x+1)(x+2)^4}$ in the usual *xy*-plane.

If an answer to a part below is none, write "NONE." (6 points total; 2 each)

a) Give the *x*-coordinate(s) of the hole(s), if any.
(Holes correspond to "removable discontinuities.")

b) Find the equation(s) of the vertical asymptote(s) (VAs), if any.

c) Find the equation of the horizontal asymptote (HA), if any.

16) Consider the graph of $y = \dfrac{3x^2+1}{6x^2-6}$ in the usual *xy*-plane. If an answer to a part below is none, write "NONE." Box in the answers! (14 points total)

a) Find the *x*-intercept(s), if any. (3 points)

b) Find the *y*-intercept, if any. (3 points)

c) Find the equation(s) of the vertical asymptote(s) (VAs), if any. (5 points)

d) Find the equation of the horizontal asymptote (HA), if any. (3 points)

17) Write the domain of *f*, where $f(x) = \dfrac{1}{\sqrt{x^2-16}}$, using interval form (the form using parentheses and/or brackets). (5 points)

18) Write the **domain** of f, where $f\left(x\right)=10^{x}$, in interval form (the form using parentheses and/or brackets). (1 point)

19) Write the **range** of f, where $f\left(x\right)=10^{x}$, in interval form (the form using parentheses and/or brackets). (1 point)

20) Write the **domain** of f, where $f\left(x\right)=\ln\left(x\right)$, in interval form (the form using parentheses and/or brackets). (1 point)

21) Write the **range** of f, where $f\left(x\right)=\ln\left(x\right)$, in interval form (the form using parentheses and/or brackets). (1 point)

22) Simplify the following: (4 points total; 2 points each)

a) $\log_{25}\left(5\right)$

b) $\log_{4}\left(\frac{1}{16}\right)$

23) Simplify: $\log_{4}\left(32\right)+\log_{4}\left(2\right)$. (3 points)

24) Expand and evaluate where appropriate: $\ln\left[\frac{e^{3}\left(\sqrt{x}\right)}{y^{2}z^{5}}\right]$. Assume $x, y, z>0$.

(10 points)

25) Find all real solution(s) of the equation: $\log(3x+1)-\log(x-1)=\log(x+2)$. Write the solution set. Show all work, as in class; do not use trial-and-error! (10 points)

26) Find the real solution of the equation: $\frac{1}{9}\left(3^{4x+1}\right)=3$. The solution is a rational number, and you must write it in simplified form. Show all work, as in class; do not use trial-and-error! (6 points)