Math 141 Name: ________________________

MIDTERM 2 – PART 1

(CHAPTERS 2 AND 3: POLYNOMIAL, RATIONAL, EXP'L, LOG FUNCTIONS)
MATH 141 – FALL 2021 – KUNIYUKI
150 POINTS TOTAL: 40 FOR PART 1, AND 110 FOR PART 2

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed.

Unless otherwise specified, give exact answers.
Write units where appropriate in your answers.

PART 1: USING SCIENTIFIC CALCULATORS (40 PTS.)

1) A profit function for a (cheap) statue-making company is given by: $p(x) = -200x^2 + 1200x - 200$ (in dollars), where x is the number of statues produced, and $x \geq 0$. Write units! (10 points total)

   a) Use a formula we used in class to find the production level (that is, the number of statues produced) that will lead to the maximum profit.

   b) What is the maximum profit?

   c) What is the profit when no statues are produced?

2) Consider $f(r) = r^3 + 5r^2 + 12r + 8$ in parts a) and b) below.
Hint: One of the zeros is -1. (16 points total)

a) Write the two other complex zeros of f in simplest, standard form.
Show all work, as in class. Box in your answers! (13 points)

b) Write the polynomial $f(r)$ as a product of three linear factors over $\mathbb{C}$, the set of complex numbers. We basically want the Linear Factorization Theorem (LFT) Form of the factorization. (3 points)

3) An exponential growth model for the population of Fredonia is given by:
$P(t) = P_0 e^{0.0471t}$, where $P(t)$ is the population t years after January 1, 2020.
The population of Fredonia was 64,000 people on January 1, 2020. (10 points)

a) In how many years after January 1, 2020 will the population of Fredonia be 200,000 people? Give **both** an **exact** answer (which may look ugly; you don't have to simplify it) and an **approximate** answer rounded off to three significant digits. Write units!

b) In what year will the population of Fredonia be 200,000 people? Use a).

4) Approximate $\log_7(902)$ to four decimal places. Show work by using a change-of-base formula we have discussed in class. (4 points)

Math 141 Name: ________________________

MIDTERM 2 – PART 2

(CHAPTERS 2 AND 3: POLYNOMIAL, RATIONAL, EXP'L, LOG FUNCTIONS)
MATH 141 – FALL 2021 – KUNIYUKI
150 POINTS TOTAL: 40 FOR PART 1, AND 110 FOR PART 2

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed.

- Unless otherwise specified, give exact answers.
- Graphs are assumed to be in the usual *xy*-plane.

PART 2: NO CALCULATORS ALLOWED! (110 PTS.)

5) Write the "Vertex Form" of the equation of the parabola in the usual *xy*-plane that has $(-1, 4)$ as its vertex and that passes through the point $(1, -16)$. (7 points)

6) Write the list of the possible rational zeros of f, where $f(x) = 5x^5 - 7x^4 - x^2 + 2$, based on the Rational Zero Test (Rational Roots Theorem). You do <u>not</u> have to determine which of these candidates are, in fact, zeros. (6 points)

7) Fill in each blank below with ∞ or $-\infty$. (4 points total; 2 points each)

a) If $f(x) = -3x^5 + 4x^2 - 1 + \dfrac{1}{x^2}$, then $\lim_{x \to -\infty} f(x) =$ ______

b) If $g(x) = 7x^4 - x$, then $\lim_{x \to \infty} g(x) =$ ______

8) Use Long Division to perform the division: $\dfrac{9x^4 - x^2 + 7x + 1}{3x^2 + 2x}$.

Write your answer in the form: $(\text{polynomial}) + (\text{proper rational expression})$.

(15 points)

9) Find a fifth-degree polynomial (with real coefficients) written in descending powers of x that has the following properties: It has $3i$ and 0 as roots (or "zeros"), and 0 is a root ("zero") of multiplicity 3.

Hint: If a polynomial with real coefficients has $3i$ as a root ("zero"), what other complex number must also be a root ("zero")? (8 points)

10) Consider $f(x) = -2x^5 + 7x^4 - x^2 + 1$. Using only Descartes's Rule of Signs, list the possible numbers of **positive** real zeros of f (accounting for multiplicity: double roots are counted twice, for example). (3 points)

11) Let $f(x) = \dfrac{2x^2 - x - 1}{x^3 - x^2}$. Consider the graph of $y = f(x)$. If an answer to a part below is none, write "NONE." Box in the answers! (19 points total)

a) Factor the numerator and the denominator of $\dfrac{2x^2 - x - 1}{x^3 - x^2}$.

b) Give the x-coordinate(s) of the hole(s), if any.
(Holes correspond to "removable discontinuities.")

c) Find the equation(s) of the vertical asymptote(s) (VAs), if any.

d) Find the equation of the horizontal asymptote (HA), if any.

e) Find the x-intercept(s), if any.

f) Find the y-intercept, if any.

12) Write the domain of f, where $f(x)=\sqrt{x^2+x-6}$, using interval form (the form using parentheses and/or brackets). (5 points)

13) Simplify $\log_8\left(\frac{1}{64}\right)$. Box in your final answer. (2 points)

14) Simplify $\log_8\left(8^{12}\right)$. Box in your final answer. (2 points)

15) Simplify $\log_{25}(5)$. Box in your final answer. (2 points)

16) Write the domain of f, where $f(x)=e^x+\ln(x)$, using interval form (the form using parentheses and/or brackets). (3 points)

17) Write the range of g, where $g(x)=e^x$, using interval form (the form using parentheses and/or brackets). (2 points)

18) Expand and evaluate where appropriate: $\log_4\left[\frac{16\left(\sqrt[3]{x}\right)}{y^2z^5}\right]$.

Assume $x, y, z > 0$. (10 points)

19) Condense: $\log(x)+3\log(y)-5\log(z)$. Assume $x, y, z>0$. (6 points)

20) Find all real solution(s) of the equation: $\log_2(x)+\log_2(x-1)=1$. Write the solution set. Show all work, as in class; do not use trial-and-error! (9 points)

21) Find all real solution(s) of the equation: $e^{2x}-7e^{x}=0$.
Write the solution set. Show all work! Hint: Factor. (7 points)