Math 141 Name: _____

MIDTERM 2 – PART 1

(CHAPTERS 2 AND 3: POLYNOMIAL, RATIONAL, EXP'L, LOG FUNCTIONS)
MATH 141 – FALL 2023 – KUNIYUKI
150 POINTS TOTAL: 48 FOR PART 1, AND 102 FOR PART 2

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed.

Unless otherwise specified, give exact answers.
Write units where appropriate in your answers.

PART 1: USING SCIENTIFIC CALCULATORS (48 PTS.)

1) An astronaut kicks a ball over a flat region of a (very) distant moon. The height of the ball in feet is given by: $h(t) = -3t^2 + 18t + 2 \ \left(\text{if } t \geq 0\right)$, where t is the amount of time in seconds since the ball was kicked. (The formula is relevant up until the time the ball hits the ground.) Write units! (19 points total)

   a) Use a formula we used in class to find how much time it takes (since the ball was kicked) for the ball to reach its maximum height. (4 points)

   b) What is the maximum height achieved by the ball? (4 points)

   c) What was the height of the ball at the time it was kicked? (3 points)

   d) How much time does it take (since the ball was kicked) for the ball to hit the ground? Give an exact answer and also round it off to three significant digits. (8 points)

2) Consider $f(t) = t^3 - 7t^2 + 17t - 14$ in parts a) and b) below.

Hint: One of the zeros is 2. (16 points total)

a) Write the two other complex zeros of f in simplest, standard form. Show all work, as in class. Box in your answers! (13 points)

b) Write the polynomial $f(t)$ as a product of three linear factors over $\mathbb{C}$, the set of complex numbers. We basically want the Linear Factorization Theorem (LFT) Form of the factorization. (3 points)

3) An exponential growth model for the population of Fredonia is given by: $P(t) = P_0 e^{0.0471t}$, where $P(t)$ is the population t years after January 1, 2010. The population of Fredonia was 64,000 people on January 1, 2010. Assume that the model is correct. (9 points total)

a) In how many years after January 1, 2010 will the population of Fredonia be 750,000 people? Give **both** an **exact** answer (which may look ugly; you don't have to simplify it) and an **approximate** answer rounded off to three significant digits. Write units! (8 points)

b) In what year will the population of Fredonia be 750,000 people? You may use part a). (1 point)

4) Approximate $\log_4(9753)$ to four decimal places. Show work by using a change-of-base formula we have discussed in class. (4 points)

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Graphs are assumed to be in the usual *xy*-plane.

PART 2: NO CALCULATORS ALLOWED! (102 POINTS)

5) Use Long Division to perform the division: $\dfrac{6x^5+4x^4-15x^2-14x}{2x^3-5}$.

Write your answer in the form: $\left(\text{polynomial}\right)+\left(\text{proper rational expression}\right)$.

(11 points)

6) Match the equations with their corresponding graphs by writing the appropriate letters in the blanks. Assume that there are no other turning (turnaround) points outside the "scope" of the figures below. The *x*- and *y*-axes are not necessarily scaled the same way within and between graphs. (8 points)

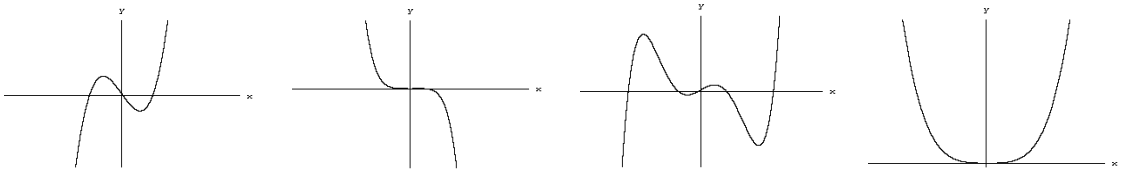
The graph of $y = x^4 + 3x^2 - x + 1$ is Graph _____.

The graph of $y = -x^5 + 2x^2 + 1$ is Graph _____.

The graph of $y = x^5 - 10x^3 + 9x + 1$ is Graph _____.

The graph of $y = x^3 - 8x + 1$ is Graph _____.

Graph A Graph B Graph C Graph D



7) Write the list of the possible rational zeros of f, where $f(x) = 2x^4 - 11x^2 + x - 3$, based on the Rational Zero Test (Rational Roots Theorem). You do not have to determine which of these candidates are, in fact, zeros. (6 points)

8) Simplify i^{502}. (2 points)

9) Let $f(x) = x^5 + 9x^3$. Write the three distinct complex zeros of f and their multiplicities in the table below. (8 points)

Zero	Multiplicity

10) Consider $f(x) = 7x^5 - 3x^4 - 2x + 5$. Using only Descartes's Rule of Signs, … (8 points total)

a) List the possible numbers of **positive** real zeros of f (accounting for multiplicity: double roots are counted twice, for example). (3 points)

b) List the possible numbers of **negative** real zeros of f (accounting for multiplicity: double roots are counted twice, for example). Show work, as in class. (5 points)

11) Write the equation of the **horizontal** asymptote (HA) for the graph of $y = \dfrac{5x^4 + 3x}{9x^4 - x^2 + 1}$ in the usual *xy*-plane. (3 points)

12) Yes or No: Does the graph of $y = \dfrac{x^2 - 36}{x - 6}$ have a vertical asymptote (VA) with equation $x = 6$ in the usual *xy*-plane? Box in one: (2 points)

Yes No

13) Consider the graph of $y = \dfrac{x - 4}{x^2 - 2x - 3}$ in the usual *xy*-plane. If an answer to a part below is none, write "NONE." Box in the answers! (14 points total)

a) Find the equation(s) of the vertical asymptote(s) (VAs), if any. (5 points)

b) Find the equation of the horizontal asymptote (HA), if any. (3 points)

c) Find the *x*-intercept(s), if any. (3 points)

d) Find the *y*-intercept, if any. (3 points)

14) Write the domain of f, where $f\left(x\right)=\sqrt[4]{x^2-16}$ using interval form (the form using parentheses and/or brackets). (5 points)

15) Let $f\left(x\right)=e^x\ln\left(x+5\right)$. Write the **domain** of f using interval form (the form using parentheses and/or brackets). (3 points)

16) Simplify the following: (6 points total; 2 points each)

a) $\log_{16}\left(2\right)$

b) $\log_8\left(8^{12}\right)$

c) $\log_3\left(\frac{1}{27}\right)$

17) Expand and evaluate where appropriate: $\log\left[\frac{x^2\left(\sqrt[3]{y}\right)}{1000z^3}\right]$. Assume $x, y, z>0$.

(10 points)

18) Find all real solution(s) of the equation: $\log_5(x) - \log_5(x-100) = 1$.
Write the solution set. Show all work, as in class; do not use trial-and-error!
(9 points)

19) Find all real solution(s) (in simplified form) of the equation: $4^{x-1} = 16^{2x}$.
Write the solution set. Show all work, as in class; do not use trial-and-error!
(7 points)