

MIDTERM 2 – PART 1

(CHAPTERS 2 AND 3: POLYNOMIAL, RATIONAL, EXP'L, LOG FUNCTIONS)

MATH 141 – FALL 2024 – KUNIYUKI

150 POINTS TOTAL: 43 FOR PART 1, AND 107 FOR PART 2

Show all work, simplify as appropriate, and use “good form and procedure” (as in class).

Box in your final answers!

No notes or books allowed.

Unless otherwise specified, give exact answers.

Write units where appropriate in your answers.**PART 1: USING SCIENTIFIC CALCULATORS (43 PTS.)**

- 1) While visiting the planet Gork, you throw a ball. The height of the ball in feet [until it hits the ground] is given by: $s(t) = -3t^2 + 24t + 5$ (if $t \geq 0$), where t is the amount of time in seconds after the ball is thrown. Write units! (12 points)
- a) Use a formula we used in class to find how much time it takes (after the ball is thrown) for the ball to reach its maximum height.
- b) What is the maximum height achieved by the ball?
- c) What was the height of the ball at the time it was thrown?
- d) If we want to know how much time it takes (after the ball is thrown) for the ball to hit the ground, what would we do? Box in one:
- Set $t = 0$ and evaluate $s(0)$. Check that $s(0) > 0$.
 - Solve $s(t) = 0$ for t , where $t > 0$.
 - Solve $s(t) = k$ for t , where $t > 0$ and k is the answer to part b).

2) Consider $g(t) = 2t^3 - 8t^2 + 9t - 2$. Hint: One of the zeros is 2. (17 points total)

a) Write the two other complex zeros of g in simplest form. Show all work, as in class. Box in your answers! Hint: The zeros are real. (13 points)

b) Write the polynomial $g(t)$ as a product of a constant and three linear factors over \mathbb{C} , the set of complex numbers. We basically want the Linear Factorization Theorem (LFT) Form of the factorization. (4 points)

3) An exponential growth model for the population of Fredonia is given by:

$$P(t) = P_0 e^{0.0471t}, \text{ where } P(t) \text{ is the population } t \text{ years after January 1, 2020.}$$

The population of Fredonia was 64,000 people on January 1, 2020. (10 points)

- a) In how many years after January 1, 2020 will the population of Fredonia be 200,000 people? Give **both** an **exact** answer (which may look ugly; you don't have to simplify it) and an **approximate** answer rounded off to three significant digits. Write units!

- b) In what year will the population of Fredonia be 200,000 people? Use a).

4) Approximate $\log_7(902)$ to four decimal places. Show work by using a change-of-base formula we have discussed in class. (4 points)

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Show all work, simplify as appropriate, and use “good form and procedure” (as in class).

Box in your final answers!

No notes or books allowed.

- Unless otherwise specified, give exact answers.
- Graphs are assumed to be in the usual xy -plane.

PART 2: NO CALCULATORS ALLOWED! (107 PTS.)

- 5) Write the “Vertex Form” of the equation of the parabola in the usual xy -plane that has $(-1, 4)$ as its vertex and that passes through the point $(1, -16)$.

(7 points)

- 6) Write the list of the possible rational zeros of f , where

$$f(x) = 5x^5 - 7x^4 - x^2 + 2, \text{ based on the Rational Zero Test (Rational Roots}$$

Theorem). You do not have to determine which of these candidates are, in fact, zeros. (6 points)

7) Fill in each blank below with ∞ or $-\infty$. (4 points total; 2 points each)

a) If $f(x) = -3x^4 + 2x - 5$, then $\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$

b) If $g(x) = 5x^3 + 2x^2 - \frac{1}{x^3}$, then $\lim_{x \rightarrow -\infty} g(x) = \underline{\hspace{2cm}}$

8) Use Long Division to perform the division: $\frac{6x^3 + 2x^2 + 15x - 2}{3x + 1}$.

Write your answer in the form: (polynomial) + (proper rational expression).

(11 points)

9) Factor $x^4 + 8x^2 + 16$ completely over \mathbb{C} , the set of complex numbers.
(4 points)

10) Fill in the blank with the correct number: In Problem 9, we can see that $x^4 + 8x^2 + 16$ has two distinct complex zeros, each with multiplicity _____.
(1 point)

- 11) How many **distinct** real zeros can a 4th-degree polynomial in x with (only) real coefficients have? Hint: Consider x -intercepts and/or factors. (3 points)
- 12) Consider $f(x) = -2x^5 + 7x^4 - x^2 + 1$. Using only Descartes's Rule of Signs, list the possible numbers of **positive** real zeros of f (accounting for multiplicity: double roots are counted twice, for example). (3 points)
- 13) Let $f(x) = \frac{2x^2 - x - 1}{x^3 - x^2}$. Consider the graph of $y = f(x)$. If an answer to a part below is none, write "NONE." Box in the answers! (19 points total)
- a) Factor the numerator and the denominator of $\frac{2x^2 - x - 1}{x^3 - x^2}$.
- b) Give the x -coordinate(s) of the hole(s), if any.
(Holes correspond to "removable discontinuities.")
- c) Find the equation(s) of the vertical asymptote(s) (VAs), if any.
- d) Find the equation of the horizontal asymptote (HA), if any.

e) Find the x -intercept(s), if any.

f) Find the y -intercept, if any.

14) Write the domain of f , where $f(x) = \sqrt{9 - x^2}$, using interval form (the form using parentheses and/or brackets). (5 points)

15) Simplify the following; box in your final answers:
(6 points total; 2 points each)

a) $\log_5\left(\frac{1}{125}\right)$

b) $\log_3(\sqrt{3})$

c) $\log_7(7^{10})$

16) Write the domain of f , where $f(x) = e^x + \ln(x)$, using interval form (the form using parentheses and/or brackets). (3 points)

17) Write the range of g , where $g(x) = e^x$, using interval form (the form using parentheses and/or brackets). (2 points)

18) Expand and evaluate where appropriate: $\log_4 \left[\frac{16(\sqrt[3]{x})}{y^2 z^5} \right]$.

Assume $x, y, z > 0$. (10 points)

19) Condense: $\log(x) + 3\log(y) - 5\log(z)$. Assume $x, y, z > 0$. (6 points)

20) Find all real solution(s) of the equation: $2\log_2(x+1) = \log_2(6x+6)$.
Write the solution set. Show all work, as in class; do not use trial-and-error!
(10 points)

- 21) Find all real solution(s) of the equation: $e^{2x} - e^x - 6 = 0$.
Write the solution set. Show all work! Hint: Factor. (7 points)