Math 141 Name: ________________________

MIDTERM 2 – PART 1

(CHAPTERS 2 AND 3: POLYNOMIAL, RATIONAL, EXP'L, LOG FUNCTIONS)
MATH 141 – SPRING 2022 – KUNIYUKI
150 POINTS TOTAL: 43 FOR PART 1, AND 107 FOR PART 2

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed.

- Unless otherwise specified, give exact answers.
- **Write units where appropriate in your answers.**

PART 1: USING SCIENTIFIC CALCULATORS (43 PTS.)

1) While visiting the planet Gork, you throw a ball. The height of the ball in feet [until it hits the ground] is given by: $s(t) = -3t^2 + 24t + 5 \ (\text{if } t \geq 0)$, where t is the amount of time in seconds after the ball is thrown. Write units! (12 points)

   a) Use a formula we used in class to find how much time it takes (after the ball is thrown) for the ball to reach its maximum height.

   b) What is the maximum height achieved by the ball?

   c) What was the height of the ball at the time it was thrown?

   d) If we want to know how much time it takes (after the ball is thrown) for the ball to hit the ground, what would we do? Box in one:

      i. Set $t = 0$ and evaluate $s(0)$. Check that $s(0) > 0$.

      ii. Solve $s(t) = 0$ for t, where $t > 0$.

      iii. Solve $s(t) = k$ for t, where $t > 0$ and k is the answer to part b).

2) Approximate $\log_8(179)$ to four decimal places. Show work by using a change-of-base formula we have discussed in class. (4 points)

3) Consider $g(t) = 2t^3 - 8t^2 + 9t - 2$. Hint: One of the zeros is 2. (17 points total)

a) Write the two other complex zeros of g in simplest form. Show all work, as in class. Box in your answers! Hint: The zeros are real. (13 points)

b) Write the polynomial $g(t)$ as a product of a constant and three linear factors over $\mathbb{C}$, the set of complex numbers. We basically want the Linear Factorization Theorem (LFT) Form of the factorization. (4 points)

4) On the day of a child's birth, a deposit of $5000 is made in a trust fund that pays 7.5% annual interest compounded continuously. Assuming there are no further deposits or withdrawals, how old will the child be when there is $8000 in the account? Give **both** an **exact** answer (which may look ugly; you don't have to simplify it) and an **approximate** answer rounded off to three significant digits. Write units! (10 points)

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PART 2: NO CALCULATORS ALLOWED! (107 PTS.)

- Unless otherwise specified, give exact answers.
- Graphs are assumed to be in the usual *xy*-plane.

5) What is the vertex of the parabola given by $y = 5(x+7)^2 - 4$? (2 points)

6) Fill in each blank below with ∞ or $-\infty$. (4 points total; 2 points each)

   a) If $f(x) = -3x^4 + 2x - 5$, then $\lim\limits_{x \to \infty} f(x) =$ ________

   b) If $g(x) = 5x^3 + 2x^2 - \dfrac{1}{x^3}$, then $\lim\limits_{x \to -\infty} g(x) =$ ________

7) Use Long Division to perform the division: $\dfrac{6x^3 + 2x^2 + 15x - 2}{3x+1}$.

   Write your answer in the form: $(\text{polynomial}) + (\text{proper rational expression})$.
   (11 points)

8) Simplify i^{403}. (2 points)

9) Write the list of the possible rational zeros of f, where $f(x) = 7x^5 - 5x^3 + 9x + 3$, based on the Rational Zero Test (Rational Roots Theorem). You do not have to determine which of these candidates are, in fact, zeros. (6 points)

10) Factor $x^4 + 8x^2 + 16$ completely over $\mathbb{C}$, the set of complex numbers. (4 points)

11) Fill in the blank with the correct number: In Problem 10, we can see that $x^4 + 8x^2 + 16$ has two distinct complex zeros, each with multiplicity _____ . (1 point)

12) How many **distinct** real zeros can a 4^{th}-degree polynomial with (only) real coefficients have? Hint: Consider x-intercepts and/or factors. (3 points)

13) Consider $f(x) = 3x^5 + 2x^4 - x^2 + 3x - 1$. Using only Descartes's Rule of Signs, … (8 points total)

a) List the possible numbers of **positive** real zeros of f (accounting for multiplicity: double roots are counted twice, for example).

b) List the possible numbers of **negative** real zeros of f (accounting for multiplicity: double roots are counted twice, for example).

14) Let $f(x) = \dfrac{x^2 - x - 6}{2x^2 + 5x + 2}$. Consider the graph of $y = f(x)$. If an answer to a part below is none, write "NONE." Box in the answers! (20 points total)

a) Factor the numerator and the denominator of $\dfrac{x^2 - x - 6}{2x^2 + 5x + 2}$, and simplify the expression. (5 points)

b) Find the x-intercept(s), if any. (3 points)

c) Find the y-intercept, if any. (3 points)

d) Give the x-coordinate(s) of the hole(s), if any.
(Holes correspond to "removable discontinuities.") (3 points)

e) Find the equation(s) of the vertical asymptote(s) (VAs), if any. (3 pts.)

f) Find the equation of the horizontal asymptote (HA), if any. (3 points)

15) Write the domain of f, where $f(x)=\sqrt{9-x^2}$, using interval form (the form using parentheses and/or brackets). (5 points)

16) Write the domain of f, where $f(x)=e^x$, using interval form (the form using parentheses and/or brackets). (1 point)

17) Write the domain of g, where $g(x)=\ln(x)$, using interval form (the form using parentheses and/or brackets). (1 point)

18) Simplify the following; box in your final answers:
(6 points total; 2 points each)

a) $\log_3\left(\sqrt{3}\right)$

b) $\log_7\left(7^{10}\right)$

c) $\log_5\left(\frac{1}{125}\right)$

19) Which of the following is equivalent to $\left[\log(x)\right]^5 + 2^{x+3}$? Box in one: (3 points)

a) $\left[\log(x)\right]^5 + 6\left(2^x\right)$

b) $\left[\log(x)\right]^5 + 8\left(2^x\right)$

c) $5\log(x) + 2^{x+3}$

20) What must be true of $\log_2(20)$? Box in one: (3 points)

a) It is between 2 and 3.

b) It is between 3 and 4.

c) It is between 4 and 5.

21) Expand and evaluate where appropriate: $\ln\left(\dfrac{x^2 y^3}{e^4 \cdot \sqrt[5]{z}}\right)$. Assume $x, y, z > 0$.

(10 points)

22) Find all real solution(s) of the equation: $2\log_2(x+1)=\log_2(6x+6)$.
Write the solution set. Show all work, as in class; do not use trial-and-error!
(10 points)

23) Find all real solution(s) of the equation: $e^{2x}-e^{x}-6=0$.
Write the solution set. Show all work! Hint: Factor. (7 points)