Math 141 Name: _____

MIDTERM 2 – PART 1

(CHAPTERS 2 AND 3: POLYNOMIAL, RATIONAL, EXP'L, LOG FUNCTIONS)
MATH 141 – SPRING 2023 – KUNIYUKI
150 POINTS TOTAL: 48 FOR PART 1, AND 102 FOR PART 2

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed.

Unless otherwise specified, give exact answers.
Write units where appropriate in your answers.

PART 1: USING SCIENTIFIC CALCULATORS (48 PTS.)

1) Write the "Vertex Form" of the equation of the parabola in the usual *xy*-plane that opens downward, that has $(-1, 4)$ as its vertex, and that passes through the point $(1, -16)$. (7 points)

2) A projectile is fired over a flat desert. The height of the projectile is given by $h(t) = -16t^2 + 40t + 5$, where t is time measured in seconds since the moment the projectile is fired. (The height formula is relevant up until the moment the projectile hits the ground.) Height is measured in feet. Write appropriate units in your answers! (10 points total)

   a) Find the height of the projectile at the moment that it is fired. (2 points)

   b) Use a formula we used in class to find the length of time it takes (since it was fired) for the projectile to reach its maximum height. (4 points)

   c) What is the corresponding maximum height? (4 points)

3) Consider $f(x) = 3x^3 - 10x^2 + 7x - 12$ in parts a) and b) below.

Hint: One of the zeros is 3. (17 points total)

a) Write the two other complex zeros of f in simplest, standard form. Show all work, as in class. Box in your answers! (13 points)

b) Write the polynomial $f(x)$ as a product of a constant and three linear factors over $\mathbb{C}$, the set of complex numbers. We basically want the Linear Factorization Theorem (LFT) Form of the factorization. (4 points)

4) On the day of a child's birth, a deposit of $2000 is made in a trust fund that pays 5.5% annual interest compounded continuously. Assuming there are no further deposits or withdrawals, how old will the child be when there is $15,000 in the account? Give **both** an **exact** answer (which may look ugly; you don't have to simplify it) and an **approximate** answer rounded off to three significant digits. Write units! (10 points)

5) Approximate $\log_5(1234)$ to four decimal places. Show work by using a change-of-base formula we have discussed in class. (4 points)

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Unless otherwise specified, give exact answers.
Graphs are assumed to be in the usual *xy*-plane.

PART 2: NO CALCULATORS ALLOWED! (102 POINTS)

6) Fill in each blank below with ∞ or $-\infty$. (4 points total; 2 points each)

a) If $f(x) = 4x^5 - 2x^4 + 3$, then $\lim_{x \to -\infty} f(x) =$ _____

b) If $g(x) = 5x^4 + x^2 - \frac{1}{x}$, then $\lim_{x \to \infty} g(x) =$ _____

7) Use Long Division to perform the division: $\frac{6x^5 + 2x^3 - 12x^2 + 4x - 5}{3x^2 + 1}$.

Write your answer in the form: $(\text{polynomial}) + (\text{proper rational expression})$.

(11 points)

8) If $f(x)$ is a nonzero polynomial with real coefficients such that one of its zeros is $3+5i$, what other complex number must also be a zero of $f(x)$? (1 point)

9) Write the list of the possible rational zeros of f, where $f(x)=7x^5+12x^3-4x^2+2$, based on the Rational Zero Test (Rational Roots Theorem). You do <u>not</u> have to determine which of these candidates are, in fact, zeros. (6 points)

10) Simplify i^{447}. (2 points)

11) Simplify $\frac{1}{2+i}$ by writing the quotient in standard form. (4 points)

12) Consider $f(x)=3x^6-7x^4+2x^3+1$. Using only Descartes's Rule of Signs, … (8 points total)

a) List the possible numbers of **positive** real zeros of f (accounting for multiplicity: double roots are counted twice, for example). (3 points)

b) List the possible numbers of **negative** real zeros of f (accounting for multiplicity: double roots are counted twice, for example). Show work, as in class. (5 points)

13) Consider the graph of $y = \dfrac{(x+1)^2(x+2)}{(x+1)(x+2)^4}$ in the usual *xy*-plane.

If an answer to a part below is none, write "NONE." Box in the answers! (6 points total; 2 each)

a) Give the *x*-coordinate(s) of the hole(s), if any.
(Holes correspond to "removable discontinuities.")

b) Find the equation(s) of the vertical asymptote(s) (VAs), if any.

c) Find the equation of the horizontal asymptote (HA), if any.

14) Consider the graph of $y = \dfrac{3x^2+1}{6x^2-6}$ in the usual *xy*-plane. If an answer to a part below is none, write "NONE." Box in the answers! (14 points total)

a) Find the *x*-intercept(s), if any. (3 points)

b) Find the *y*-intercept, if any. (3 points)

c) Find the equation(s) of the vertical asymptote(s) (VAs), if any. (5 points)

d) Find the equation of the horizontal asymptote (HA), if any. (3 points)

15) Write the domain of *f*, where $f(x) = \sqrt{x^2-4x+3}$ using interval form (the form using parentheses and/or brackets). (5 points)

16) Write the **domain** of f, where $f\left(x\right)=10^{x}$, in interval form (the form using parentheses and/or brackets). (1 point)

17) Write the **range** of f, where $f\left(x\right)=10^{x}$, in interval form (the form using parentheses and/or brackets). (1 point)

18) Write the **domain** of f, where $f\left(x\right)=\ln\left(x\right)$, in interval form (the form using parentheses and/or brackets). (1 point)

19) Write the **range** of f, where $f\left(x\right)=\ln\left(x\right)$, in interval form (the form using parentheses and/or brackets). (1 point)

20) Simplify the following: (6 points total; 2 points each)

a) $\log_{25}\left(5\right)$

b) $\log_{4}\left(\frac{1}{16}\right)$

c) $4^{\log_{4}\left(20\right)}$

21) Simplify: $\log_{4}\left(32\right)+\log_{4}\left(2\right)$. (4 points)

22) Expand and evaluate where appropriate: $\ln\left[\dfrac{e^3\left(\sqrt{x}\right)}{y^2z^5}\right]$. Assume $x, y, z > 0$.

(10 points)

23) Find all real solution(s) of the equation: $\log_2(x) + \log_2(x+2) = 3$.

Write the solution set. Show all work, as in class; do not use trial-and-error!
(10 points)

24) Find the real solution of the equation: $e^{2x} - 11e^{x} = 0$.
Write the solution set. Show all work! Hint: Factor. (7 points)