Math 141 Name: _____

MIDTERM 2 – PART 1

(CHAPTERS 2 AND 3: POLYNOMIAL, RATIONAL, EXP'L, LOG FUNCTIONS)
MATH 141 – SPRING 2024 – KUNIYUKI
150 POINTS TOTAL: 45 FOR PART 1, AND 105 FOR PART 2

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed.

Unless otherwise specified, give exact answers.
Write units where appropriate in your answers.

PART 1: USING SCIENTIFIC CALCULATORS (45 PTS.)

1) A profit function for a (cheap) statue-making company is given by: $p(x) = -200x^2 + 1200x - 200$ (in dollars), where x is the number of statues produced, and $x \geq 0$. Write units! (10 points total)

   a) Use a formula we used in class to find the production level (that is, the number of statues produced) that will lead to the maximum profit. (4 points)

   b) What is the maximum profit? (4 points)

   c) What is the profit when no statues are produced? (2 points)

2) Simplify $\frac{1}{3-7i}$ by writing the quotient in standard form. (5 points)

3) Consider $f(r) = r^3 + 5r^2 + 12r + 8$ in parts a) and b) below.

Hint: One of the zeros is -1. (16 points total)

a) Write the two other complex zeros of f in simplest, standard form. Show all work, as in class. Box in your answers! (13 points)

b) Write the polynomial $f(r)$ as a product of three linear factors over $\mathbb{C}$, the set of complex numbers. We basically want the Linear Factorization Theorem (LFT) Form of the factorization. (3 points)

4) On the day of a child's birth, a deposit of $2000 is made in a trust fund that pays 5.5% annual interest compounded continuously. Assuming there are no further deposits or withdrawals, how old will the child be when there is $15,000 in the account? Give **both** an **exact** answer (which may look ugly; you don't have to simplify it) and an **approximate** answer rounded off to three significant digits. Write units! (10 points)

5) Approximate $\log_5(1684)$ to four decimal places. Show work by using a change-of-base formula we have discussed in class. (4 points)

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Unless otherwise specified, give exact answers.
Graphs are assumed to be in the usual *xy*-plane.

PART 2: NO CALCULATORS ALLOWED! (105 POINTS)

6) The equation of a parabola is given by: $y = -7(x-4)^2 + 5$. (4 points total)

   a) Which way does the parabola open? Box in one: Upward Downward

   b) What is the vertex of the parabola?

7) Fill in each blank below with ∞ or $-\infty$. (4 points total; 2 points each)

   a) If $f(x) = -3x^5 + 4x^2 - 1 + \frac{1}{x^2}$, then $\lim_{x \to -\infty} f(x) =$ _____

   b) If $g(x) = 7x^4 - x$, then $\lim_{x \to \infty} g(x) =$ _____

8) Write the list of the possible rational zeros of f, where $f(x) = 2x^5 + 3x^3 - 6x^2 + 7$, based on the Rational Zero Test (Rational Roots Theorem). You do <u>not</u> have to determine which of these candidates are, in fact, zeros. (6 points)

9) Use Long Division to perform the division: $\dfrac{9x^4 - x^2 + 7x + 1}{3x^2 + 2x}$.

Write your answer in the form: $\left(\text{polynomial}\right) + \left(\text{proper rational expression}\right)$.

(15 points)

10) Simplify i^{403}. (3 points)

11) Consider $f\left(x\right) = 3x^5 + 2x^4 - x^2 + 3x - 1$. Using only Descartes's Rule of Signs, … (8 points total)

a) List the possible numbers of **positive** real zeros of f (accounting for multiplicity: double roots are counted twice, for example). (3 points)

b) List the possible numbers of **negative** real zeros of f (accounting for multiplicity: double roots are counted twice, for example).
Show work, as in class. (5 points)

12) Factor $3x^4 - 75$ as a product of a constant and four linear factors over $\mathbb{C}$, the set of complex numbers. We basically want the Linear Factorization Theorem (LFT) Form of the factorization. (8 points)

13) Consider the graph of $y = f(x)$, where $f(x) = \dfrac{3x^2 + 2x - 1}{9x^2 - 4}$. If an answer to a part below is none, write "NONE." Box in the answers! (20 points total)

a) Factor the numerator and the denominator of $\dfrac{3x^2 + 2x - 1}{9x^2 - 4}$. (4 points)

b) Yes or No: Does the graph of $y = f(x)$ have any holes? (Holes correspond to "removable discontinuities.") Box in one: (2 points)

Yes No

c) Find the equation(s) of the vertical asymptote(s) (VAs), if any. (4 points)

d) Find the equation of the horizontal asymptote (HA), if any. (3 points)

e) Find the *x*-intercept(s), if any. (4 points)

f) Find the *y*-intercept, if any. (3 points)

14) Write the domain of f, where $f(x)=\sqrt{x^2+x-6}$, using interval form (the form using parentheses and/or brackets). (5 points)

15) Simplify the following; box in your final answers: (6 points total; 2 each)

a) $\log_{25}(5)$

b) $\log(1000)$

c) $\log_2\left(\frac{1}{8}\right)$

16) Expand and evaluate where appropriate: $\ln\left[\frac{e^4x^5}{(y^3)(\sqrt{z})}\right]$.

Assume $x, y, z > 0$. (10 points)

17) Find all real solution(s) of the equation: $\log_2(x)+\log_2(x-1)=1$. Write the solution set. Show all work, as in class; do <u>not</u> use trial-and-error! (9 points)

18) Find all real solution(s) of the equation: $e^{2x}-7e^{x}=0$.
Write the solution set. Show all work! Hint: Factor. (7 points)