

MIDTERM 2 – PART 1**(CHAPTERS 2 AND 3: POLYNOMIAL, RATIONAL, EXP'L, LOG FUNCTIONS)****MATH 141 – SPRING 2025 – KUNIYUKI****150 POINTS TOTAL: 44 FOR PART 1, AND 106 FOR PART 2****Show all work, simplify as appropriate, and use “good form and procedure” (as in class).****Box in your final answers!****No notes or books allowed.**

- Unless otherwise specified, give exact answers.
- **Write units where appropriate in your answers.**

PART 1: USING SCIENTIFIC CALCULATORS (44 PTS.)

- 1) The profit P (in dollars) for the Superdoom computer game company is given by P or $P(x) = -20x^2 + 400x - 1500$, where x is the number of game DVDs produced and sold. You may assume that the domain of P is $[0, \infty)$.

For parts a) and b), write units! (8 points total)

- a) Write and use a formula we used in class to find the number of DVDs (produced and sold) for which profit is maximized. (4 points)

- b) What is the corresponding maximum profit? (4 points)

2) Consider $s(r) = r^3 - 3r^2 - 4r + 42$ in parts a) and b) below.

Hint: One of the zeros is -3 . (16 points total)

a) Write the two other complex zeros of s in simplest, standard form. Show all work, as in class. Box in your answers! (13 points)

b) Write the polynomial $s(r)$ as a product of three linear factors over \mathbb{C} , the set of complex numbers. We basically want the Linear Factorization Theorem (LFT) Form of the factorization. (3 points)

3) On the day of a child's birth, a deposit of \$5000 is made in a trust fund that pays 7.5% annual interest compounded continuously. Assuming there are no further deposits or withdrawals, how old will the child be when there is \$8000 in the account? Give **both** an **exact** answer (which may look ugly; you don't have to simplify it) and an **approximate** answer rounded off to three significant digits. Write units! (10 points)

4) Approximate $\log_8(179)$ to four decimal places. Show work by using a change-of-base formula we have discussed in class. (4 points)

5) Simplify $\frac{1}{2-7i}$ by writing the quotient in standard form. (6 points)

MIDTERM 2 – PART 2

(CHAPTERS 2 AND 3: POLYNOMIAL, RATIONAL, EXP'L, LOG FUNCTIONS)

MATH 141 – SPRING 2025 – KUNIYUKI

150 POINTS TOTAL: 44 FOR PART 1, AND 106 FOR PART 2

Show all work, simplify as appropriate, and use “good form and procedure” (as in class).

Box in your final answers!

No notes or books allowed.

PART 2: NO CALCULATORS ALLOWED! (106 PTS.)

- Unless otherwise specified, give exact answers.
- Graphs are assumed to be in the usual xy -plane.

6) What is the vertex of the parabola given by $y = 5(x + 7)^2 - 4$? (2 points)7) Fill in each blank below with ∞ or $-\infty$. (4 points total; 2 points each)

a) If $f(x) = -2x^3 + 5x^2 - 3 + \frac{1}{x^2}$, then $\lim_{x \rightarrow \infty} f(x) =$ _____

b) If $g(x) = 2x^4 + 3x^2 + 2$, then $\lim_{x \rightarrow -\infty} g(x) =$ _____

8) How many turning points (TPs) can the graph of $y = f(x)$ have if f is a 4th-degree polynomial function? (3 points)9) Simplify i^{447} . (2 points)

10) Write the list of the possible rational zeros of f , where

$f(x) = 7x^5 - 5x^3 + 9x + 3$, based on the Rational Zero Test (Rational Roots Theorem). You do not have to determine which of these candidates are, in fact, zeros. (6 points)

11) Use Long Division to perform the division: $\frac{8x^4 + 12x^3 - 6x^2 - 7x + 1}{4x^2 - 3}$.

Write your answer in the form: (polynomial) + (proper rational expression).
(11 points)

- 12) Find a fifth-degree polynomial (with real coefficients) written in descending powers of x that has the following properties: It has $3i$ and 0 as roots (or “zeros”), and 0 is a root (“zero”) of multiplicity 3.

Hint: If a polynomial with real coefficients has $3i$ as a root (“zero”), what other complex number must also be a root (“zero”)? (8 points)

- 13) Consider $f(x) = 4x^7 + x^3 - 2x^2 + 9x - 3$. Using only Descartes’s Rule of Signs, ... (8 points total)

- a) List the possible numbers of **positive** real zeros of f (accounting for multiplicity: double roots are counted twice, for example).

- b) List the possible numbers of **negative** real zeros of f (accounting for multiplicity: double roots are counted twice, for example).

14) Let $f(x) = \frac{x^2 - x - 6}{2x^2 + 5x + 2}$. Consider the graph of $y = f(x)$. If an answer to a part below is none, write "NONE." Box in the answers! (20 points total)

a) Factor the numerator and the denominator of $\frac{x^2 - x - 6}{2x^2 + 5x + 2}$, and simplify the expression. (5 points)

b) Find the x -intercept(s), if any. (3 points)

c) Find the y -intercept, if any. (3 points)

d) Give the x -coordinate(s) of the hole(s), if any.
(Holes correspond to "removable discontinuities.") (3 points)

e) Find the equation(s) of the vertical asymptote(s) (VAs), if any. (3 pts.)

f) Find the equation of the horizontal asymptote (HA), if any. (3 points)

15) Write the domain of f , where $f(x) = \frac{1}{\sqrt{x^2 - 16}}$, using interval form (the form using parentheses and/or brackets). (5 points)

16) Write the domain of f , where $f(x) = e^x$, using interval form (the form using parentheses and/or brackets). (1 point)

17) Write the domain of g , where $g(x) = \ln(x)$, using interval form (the form using parentheses and/or brackets). (1 point)

18) Simplify the following: (4 points total; 2 points each)

a) $\log\left(\frac{1}{100}\right)$

b) $\log_8(2)$

19) Which of the following is equivalent to $[\log(x)]^5 + 2^{x+3}$? Box in one: (3 points)

a) $[\log(x)]^5 + 6(2^x)$

b) $[\log(x)]^5 + 8(2^x)$

c) $5\log(x) + 2^{x+3}$

20) What must be true of $\log_2(20)$? Box in one: (3 points)

a) It is between 2 and 3.

b) It is between 3 and 4.

c) It is between 4 and 5.

21) Expand and evaluate where appropriate: $\ln\left(\frac{x^2 y^3}{e^4 \cdot \sqrt[5]{z}}\right)$. Assume $x, y, z > 0$.

(10 points)

22) Find all real solution(s) of the equation: $\log(3x + 1) - \log(x - 1) = \log(x + 2)$.
Write the solution set. Show all work, as in class; do not use trial-and-error!
(10 points)

23) Find the real solution of the equation: $\frac{1}{9}(3^{4x+1}) = 3$. The solution is a rational number, and you must write it in simplified form. Show all work, as in class; do not use trial-and-error! (5 points)