

# MIDTERM 4 – PART 1

(CHAPTERS 5 AND 6: ANALYTIC & MISC. TRIGONOMETRY)

MATH 141 – FALL 2016 – KUNIYUKI

150 POINTS TOTAL: 47 FOR PART 1, AND 103 FOR PART 2

Show all work, simplify as appropriate, and use “good form and procedure” (as in class).

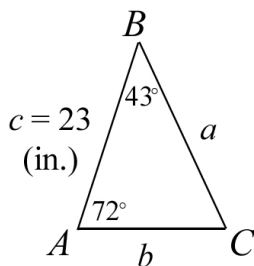
**Box in your final answers!**

**No notes or books allowed.**

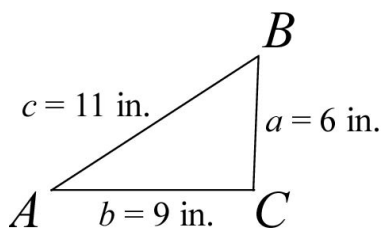
- Write units where appropriate in your final answers.
- Try to avoid rounding intermediate results; if you do round off, do it to at least five significant digits.
- We assume that all vectors on this test are in the usual  $xy$ -plane.

## PART 1: USING SCIENTIFIC CALCULATORS (47 PTS.)

- 1) Find the length of Side  $b$  for the triangle below using the Law of Sines. Round off your answer to the nearest tenth (that is, to one decimal place) of an inch. (7 points)



- 2) Two beetles crawl out of a small hole in a wall (represented by Point  $B$  below) and then crawl along linear paths on the wall. When one beetle has crawled 6 inches on the wall and the other beetle has crawled 11 inches on the wall, they are 9 inches apart. Find the angle between the beetles' paths. In other words, find the measure of Angle  $B$  for the triangle below. Use the Law of Cosines. Round off your answer to the nearest tenth of a degree. Note: Angle  $C$  is obtuse, not right. (10 points)



For the rest of Part 1, assume that  $x$  and  $y$  are scaled in meters.

3) Find the  $\langle x, y \rangle$  component form of the vector  $\mathbf{v}$  that has magnitude 15 meters and direction angle  $38^\circ$ . Round off the  $x$  and  $y$  components to the nearest tenth of a meter. (5 points)

4) Let  $\mathbf{v}$  be the vector  $4\mathbf{i} + 5\mathbf{j}$ . (9 points total)

a) Find the unit vector in the direction of  $\mathbf{v}$ . Write it in  $\langle x, y \rangle$  component form. Give an **exact** answer; do **not** approximate! Rationalize denominators in your answer. (5 points)

b) Find the direction angle of  $\mathbf{v}$ . Round off your answer to the nearest tenth of a degree. (4 points)

5) Assume that  $\mathbf{v}$  is a vector in the plane such that  $\mathbf{v} \bullet \mathbf{v} = 16$ . Find  $\|\mathbf{v}\|$ . (2 points)

6) Assume that  $\mathbf{v}$  and  $\mathbf{w}$  are two vectors in the real plane. (4 points total)

a)  $(\mathbf{v} \bullet \mathbf{w})\mathbf{w}$  is ... (Box in one:)      a scalar      a vector      (Neither)

b)  $(\mathbf{v} \bullet \mathbf{w}) \bullet \mathbf{w}$  is ... (Box in one:)      a scalar      a vector      (Neither)

7) Consider the vectors  $\mathbf{v}$  and  $\mathbf{w}$ , where  $\mathbf{v} = \langle -3, 5 \rangle$  and  $\mathbf{w} = \langle 2, -6 \rangle$ . Find the angle between  $\mathbf{v}$  and  $\mathbf{w}$  using the formula given in class. Round off your answer to the nearest tenth of a degree. (10 points)

## MIDTERM 4 – PART 2

(CHAPTERS 5 AND 6: ANALYTIC & MISC. TRIGONOMETRY)

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150 POINTS TOTAL: 47 FOR PART 1, AND 103 FOR PART 2

Show all work, simplify as appropriate, and use “good form and procedure” (as in class).

Box in your final answers!

No notes or books allowed.

### PART 2: NO CALCULATORS ALLOWED! (103 PTS.)

- 8) Complete the Identities. Fill out the table below so that, for each row, the left side is equivalent to the right side, based on the type of identity (ID) given in the last column. You do not have to show work. (18 points total)

Left Side	Right Side	Type of Identity (ID)
$\sin(u + v)$		Sum ID
$\cos(u + v)$		Sum ID
$\tan(u + v)$		Sum ID
$\sin(u - v)$		Difference ID
$\sin(2u)$		Double-Angle ID
$\tan(2u)$		Double-Angle ID
$\sin^2(u)$		Power-Reducing ID (PRI)
$\cos^2(u)$		Power-Reducing ID (PRI)
$\cos\left(\frac{\theta}{2}\right)$		Half-Angle ID

- 9) Write the three Half-Angle Identities for  $\tan\left(\frac{\theta}{2}\right)$ , as given in class. (4 points)

- 10) Simplify:  $\cos^4(x) - \sin^4(x)$ . Your final answer will be one of the following:  $\sin(2x)$ ,  $\cos(2x)$ ,  $\sin^2(2x)$ , or  $\cos^2(2x)$ . Hint: You must factor; show work. (4 points)

Make sure to show all steps, as in class!

11) Verify the identity:  $\sqrt{\frac{1 - \sin(\theta)}{1 + \sin(\theta)}} = \frac{1 - \sin(\theta)}{|\cos(\theta)|}$ . (7 points)

12) Verify the identity:  $\frac{1 + \csc(-\theta)}{\cot(\theta)} = \tan(\theta) - \sec(\theta)$ . (9 points)

- 13) Use the trigonometric substitution  $x = 7 \tan(\theta)$  to rewrite the algebraic expression  $\sqrt{x^2 + 49}$  as a trigonometric expression in  $\theta$ , where  $\theta$  is acute. Simplify! (7 points)

**WHEN SOLVING THE TRIGONOMETRIC EQUATIONS IN #14, 15, AND 16, GIVE FINAL ANSWERS EXACTLY IN RADIANS, NOT DEGREES.**

- 14) Find all real solutions of the equation:  $3\cos^2(x) - 5\cos(x) - 2 = 0$ .  
Leave your answers in exact form; do not approximate. (7 points)

15) Find all real solutions of the equation:  $\cot^2(3x) = 3$ . (9 points)

16) Use 15) to find the real solutions of  $\cot^2(3x) = 3$  in the interval  $[0, \pi]$ .

Note: This is easier than dealing with the interval  $[0, 2\pi)$ . (6 points)

17) Use a Half-Angle Identity to evaluate and simplify  $\sin\left(\frac{\pi}{8}\right)$ , as in class.  
(6 points)

18) Rewrite  $\sin(2\arcsin(x))$  as an equivalent algebraic expression.  
Assume  $x$  is in  $[-1, 1]$ . (7 points)



19) Simplify and evaluate:  $2 \cos^2\left(\frac{\pi}{12}\right) - 1$ . (4 points)

20) Use **one** of the Product-to-Sum Identities below to rewrite the expression  $\sin(7x)\cos(3x)$ . Simplify. (4 points)

$$\sin(u)\sin(v) = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$$

$$\cos(u)\cos(v) = \frac{1}{2}[\cos(u-v) + \cos(u+v)]$$

$$\sin(u)\cos(v) = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$$

$$\cos(u)\sin(v) = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$$

21) Assume that  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  are nonzero vectors in the real plane.

Your answers to a), b), and c) below will be angles in the interval  $[0^\circ, 180^\circ]$ .

(9 points total; 3 points each)

a) If  $\mathbf{v} \cdot \mathbf{w} = 0$ , what is the angle between  $\mathbf{v}$  and  $\mathbf{w}$ ?

b) What is the angle between  $\mathbf{u}$  and  $4\mathbf{u}$ ?

c) What is the angle between  $\mathbf{u}$  and  $-3\mathbf{u}$ ?

22) A triangle has one side of length  $a$  and another side of length  $b$ , and Angle  $C$  is the included angle between the two sides. Give the formula we gave in class for the area of this triangle in terms of  $C$ ,  $a$ , and  $b$ . (2 points)