Math 141 Name: ________________________

MIDTERM 4 – PART 1

(CHAPTERS 5 AND 6: ANALYTIC & MISC. TRIGONOMETRY)
MATH 141 – FALL 2024 – KUNIYUKI
150 POINTS TOTAL: 50 FOR PART 1, AND 100 FOR PART 2

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed.

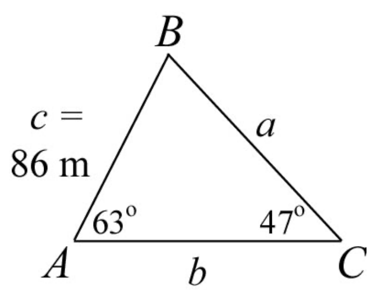
• Write units in your final answers where appropriate.

• Try to avoid rounding intermediate results; if you do round off, do it to at least five significant digits.

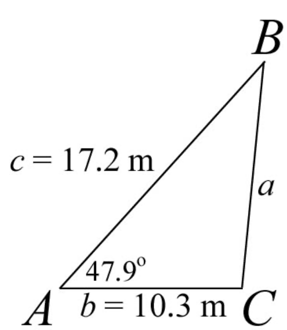
• We assume that all vectors on this test are in the usual *xy*-plane.

PART 1: USING SCIENTIFIC CALCULATORS (50 PTS.)

1) Find the length of Side b for the triangle below using the Law of Sines. Round off your answer to the nearest tenth (that is, to one decimal place) of a meter. (7 points)



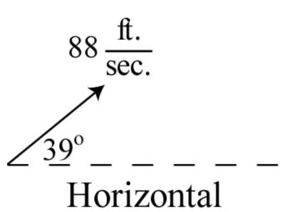
2) A tilting flagpole is represented by line segment $\overline{BC}$ in the figure below. An observer stands at point A. The observer's shoes are 10.3 meters from (and are level with) the base of the pole and are 17.2 meters from the top of the pole. The angle of elevation from the observer's shoes to the top of the pole is 47.9°. Find the length of the flagpole, which is given by a, the length of $\overline{BC}$. Use the Law of Cosines. Round off your answer to the nearest tenth (that is, to one decimal place) of a meter. Note: Angle C is obtuse, not right. (7 points)



3) Find the vector of magnitude 9 meters in the **opposite** direction of the vector **v**, where $\mathbf{v} = -5\mathbf{i} + 2\mathbf{j}$. Write the vector in $\langle x, y \rangle$ component form. Give an **exact** answer; do **not** approximate! Rationalize denominators in your answer. Assume that x and y are scaled in meters. (8 points)

4) Find the direction angle of the vector **v**, where $\mathbf{v} = -2\mathbf{i} + 3\mathbf{j}$. Round off your answer to the nearest tenth (that is, to one decimal place) of a degree. (5 points)

5) A ball is thrown with an initial velocity of 88 feet per second, at an angle of elevation of 39° from the horizontal (see figure below). Round off your answers to three significant digits and write units. (10 points total; 5 points each)

	a) What is the ball's horizontal component of initial velocity? b) What is the ball's vertical component of initial velocity?

6) Assume that **v** is a vector in the real plane.

$\|\mathbf{v}\|\mathbf{v}$ is … (Box in one:) a scalar a vector (neither)
(3 points)

7) Consider the vectors **v** and **w**, where $\mathbf{v} = \langle -3, 5 \rangle$ and $\mathbf{w} = \langle 2, -6 \rangle$. Find the angle between **v** and **w** using the formula given in class. Round off your answer to the nearest tenth (that is, to one decimal place) of a degree. Assume distance is in meters. (10 points)

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MIDTERM 4 – PART 2

(CHAPTERS 5 AND 6: ANALYTIC & MISC. TRIGONOMETRY)
MATH 141 – FALL 2024 – KUNIYUKI
150 POINTS TOTAL: 50 FOR PART 1, AND 100 FOR PART 2
Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed.

PART 2: NO CALCULATORS ALLOWED! (100 POINTS)

8) Complete the Identities. Fill out the table below so that, for each row, the left side is equivalent to the right side, based on the type of identity (ID) given in the last column. You do not have to show work. (14 points total)

Left Side	Right Side	Type of Identity (ID)
$\sin(u+v)$		Sum ID
$\cos(u+v)$		Sum ID
$\tan(u+v)$		Sum ID
$\sin(2u)$		Double-Angle ID
$\cos(2u)$		Double-Angle ID (write <u>any</u> one of the three versions we've discussed)
$\cos^2(u)$		Power-Reducing ID (PRI)
$\cos\left(\frac{\theta}{2}\right)$		Half-Angle ID

9) Write the three Half-Angle Identities for $\tan\left(\frac{\theta}{2}\right)$, as given in class. (3 points)

From now on, make sure to show all steps, as in class!

10) Verify the identity:

$\csc^4(x)\cot(x) - \csc^2(x)\cot(x) = \csc^2(x)\cot^3(x)$. (6 points)

11) Simplify: $\dfrac{1-\sin(x)}{\csc(x)-1}$. Your final answer will be one of the following:

$\sin(x)$, $\cos(x)$, $\tan(x)$, $\csc(x)$, $\sec(x)$, or $\cot(x)$. Show all work! (7 points)

12) Simplify: $\cos^4(x) - \sin^4(x)$. Your final answer will be one of the following: $\sin(2x)$, $\cos(2x)$, $\sin^2(2x)$, or $\cos^2(2x)$. Hint: You must factor; show work. (4 points)

13) Use the trigonometric substitution $x = 4\sec(\theta)$ to rewrite the algebraic expression $\sqrt{x^2 - 16}$ as a trigonometric expression in θ, where θ is acute. Show all work, as in class, and simplify. (7 points)

WHEN SOLVING THE FOLLOWING TRIGONOMETRIC EQUATIONS, GIVE FINAL ANSWERS EXACTLY IN RADIANS, NOT DEGREES.

14) Consider the equation: $2\sin(4x)-1=0$. (18 points total)

a) Find all real solutions of the equation: $2\sin(4x)-1=0$.

b) Use part a) to find the real solutions of the equation $2\sin(4x)-1=0$ in the interval $[0, 2\pi)$. You do not have to use set notation, but make sure to box in all solutions.

15) Find <u>all</u> real solutions of the equation: $\tan^2(x) - 4\tan(x) + 3 = 0$.
Write your solution(s) as **exact** expressions; do **not** approximate! (10 points)

16) Use **one** of the Product-to-Sum Identities below to rewrite the expression $\sin(7\theta)\cos(\theta)$. Simplify. (4 points)

$$\sin(u)\sin(v) = \frac{1}{2}\left[\cos(u-v) - \cos(u+v)\right]$$

$$\cos(u)\cos(v) = \frac{1}{2}\left[\cos(u-v) + \cos(u+v)\right]$$

$$\sin(u)\cos(v) = \frac{1}{2}\left[\sin(u+v) + \sin(u-v)\right]$$

$$\cos(u)\sin(v) = \frac{1}{2}\left[\sin(u+v) - \sin(u-v)\right]$$

17) Rewrite $\sin\left(2\arccos\left(x\right)\right)$ as an equivalent algebraic expression.
Assume x is in $\left[-1, 1\right]$. Show work, as in class. (7 points)

18) Use a Double-Angle Identity to simplify: $6\sin\left(5x\right)\cos\left(5x\right)$. (4 points)

19) Simplify: $\sin^2(15^\circ)$. Hint: Use a Power-Reducing ID (PRI). Make sure your final answer is not a compound fraction. (5 points)

20) A triangle has one side of length *a* and another side of length *b*, and Angle *C* is the included angle between the two sides. Give the formula we gave in class for the area of this triangle in terms of *C*, *a*, and *b*. (2 points)

21) Assume that **u**, **v**, and **w** are nonzero vectors in the real plane. Your answers to a), b), and c) below will be angles in the interval $\left[0^\circ, 180^\circ\right]$. (9 points total; 3 points each)

a) If $\mathbf{v} \bullet \mathbf{w} = 0$, what is the angle between **v** and **w**?

b) What is the angle between **u** and $4\mathbf{u}$?

c) What is the angle between **u** and $-3\mathbf{u}$?