Math 141 Name: ________________________

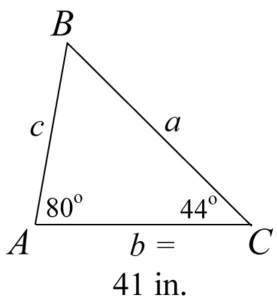
MIDTERM 4 – PART 1

(CHAPTERS 5 AND 6: ANALYTIC & MISC. TRIGONOMETRY)
MATH 141 – SPRING 2019 – KUNIYUKI
150 POINTS TOTAL: 50 FOR PART 1, AND 100 FOR PART 2
Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed.

- Write units where appropriate in your final answers.
- Try to avoid rounding intermediate results; if you do round off, do it to at least five significant digits.
- We assume that all vectors on this test are in the usual *xy*-plane.

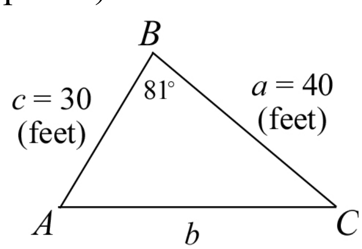
PART 1: USING SCIENTIFIC CALCULATORS (50 PTS.)

1) Find the length of Side c for the triangle below using the Law of Sines. Round off your answer to the nearest tenth (that is, to one decimal place) of an inch. (7 points)



2) Two runners start at a flagpole at Point B and run along straight lines. The angle between their paths is 81°. At noon, one runner is 40 feet away from Point B, and the other runner is 30 feet away from Point B. What is the distance between the two runners at noon? Round off your answer to the nearest tenth (that is, to one decimal place) of a foot.

   Hint: Find b, the length of line segment $\overline{AC}$, for the triangle below using the Law of Cosines. Points A and C represent the positions of the runners at noon. (7 points)



For the rest of Part 1, assume that x and y are scaled in meters.

3) For parts a), b), and c) below, consider the vectors **v** and **w**, where $\mathbf{v} = \langle 7, 2 \rangle$ and $\mathbf{w} = \langle -3, 12 \rangle$. (9 points total)

   a) Find the vector $(\mathbf{v} \bullet \mathbf{w})\mathbf{w}$. Write your answer in $\langle x, y \rangle$ component form. (5 points)

   b) Does $(\mathbf{v} \bullet \mathbf{w})\mathbf{w}$ have the same direction as the vector **w**?
   Box in one: Yes No (2 points)

   c) Are **v** and **w** orthogonal? Box in one: Yes No (2 points)

4) A vector **v** is drawn from the initial point $(-1, 4)$ to the terminal point $(-4, 11)$ in the usual *xy*-plane. Find the unit vector in the direction of this vector; write it in $\langle x, y \rangle$ component form. Give an **exact** answer; do **not** approximate! Rationalize denominators in your answer. (8 points)

5) Find the $\langle x, y \rangle$ component form of the vector **v** that has magnitude 6 meters and direction angle 77°. Round off the x and y components to the nearest hundredth (that is, to two decimal places) of a meter. (5 points)

6) Find the direction angle of the vector **v**, where $\mathbf{v} = 5\mathbf{i} + 2\mathbf{j}$. Round off your answer to the nearest tenth (that is, to one decimal place) of a degree. (4 points)

7) Consider the vectors **v** and **w**, where $\mathbf{v} = \langle -3, 5 \rangle$ and $\mathbf{w} = \langle 2, -6 \rangle$. Find the angle between **v** and **w** using the formula given in class. Round off your answer to the nearest tenth (that is, to one decimal place) of a degree. (10 points)

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MIDTERM 4 – PART 2

(CHAPTERS 5 AND 6: ANALYTIC & MISC. TRIGONOMETRY)
MATH 141 – SPRING 2019 – KUNIYUKI
150 POINTS TOTAL: 50 FOR PART 1, AND 100 FOR PART 2
Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed.

PART 2: NO CALCULATORS ALLOWED! (100 PTS.)

8) Complete the Identities. Fill out the table below so that, for each row, the left side is equivalent to the right side, based on the type of identity (ID) given in the last column. You do not have to show work. (16 points total)

Left Side	Right Side	Type of Identity (ID)
$\sin(u+v)$		Sum ID
$\cos(u+v)$		Sum ID
$\tan(u+v)$		Sum ID
$\cos(u-v)$		Difference ID
$\sin(2u)$		Double-Angle ID
$\sin^2(u)$		Power-Reducing ID (PRI)
$\cos^2(u)$		Power-Reducing ID (PRI)
$\sin\left(\frac{\theta}{2}\right)$		Half-Angle ID

9) Write the three versions of the Double-Angle Identities for $\cos(2u)$ that we discussed in class; these were listed on the "More Trig Identities" handout. (6 points total)

10) Write the three Half-Angle Identities for $\tan\left(\frac{\theta}{2}\right)$, as given in class. (3 pts.)

Make sure to show all steps, as in class!

11) Verify the identity: $\dfrac{1}{\sin(x)} + \sin(-x) = \cos(x)\cot(x)$. (11 points)

12) Verify the identity: $\dfrac{1}{1+\sin(x)} = \sec^2(x) - \sec(x)\tan(x)$. (12 points)

13) Use the trigonometric substitution $x = 4\sec(\theta)$ to rewrite the algebraic expression $\sqrt{x^2 - 16}$ as a trigonometric expression in θ, where θ is acute. Simplify! (7 points)

WHEN SOLVING THE FOLLOWING TRIGONOMETRIC EQUATIONS, GIVE FINAL ANSWERS EXACTLY IN RADIANS, NOT DEGREES.

14) Consider the equation: $2\sin(4x)-\sqrt{3}=0$. (18 points total)

a) Find <u>all</u> real solutions of the equation: $2\sin(4x)-\sqrt{3}=0$.

b) Use part a) to find the real solutions of the equation $2\sin(4x)-\sqrt{3}=0$ <u>in the interval</u> $[0, 2\pi)$. You do <u>not</u> have to use set notation, but make sure to box in all your solutions.

15) Find <u>all</u> real solutions of the equation: $\cos(2x)+\cos(x)=0$. (11 points)

16) Use a Double-Angle Identity to simplify: $10\sin(x)\cos(x)$. (3 points)

17) Use **one** of the Product-to-Sum Identities below to rewrite the expression $\cos(7\theta)\cos(4\theta)$. Simplify. (4 points)

$$\sin(u)\sin(v)=\frac{1}{2}\left[\cos(u-v)-\cos(u+v)\right]$$

$$\cos(u)\cos(v)=\frac{1}{2}\left[\cos(u-v)+\cos(u+v)\right]$$

$$\sin(u)\cos(v)=\frac{1}{2}\left[\sin(u+v)+\sin(u-v)\right]$$

$$\cos(u)\sin(v)=\frac{1}{2}\left[\sin(u+v)-\sin(u-v)\right]$$

18) A triangle has one side of length a and another side of length b, and Angle C is the included angle between the two sides. Give the formula we gave in class for the area of this triangle in terms of C, a, and b. (3 points)

19) Assume that **v** and **w** are two vectors in the real plane. (6 points; 3 each)

a) $(\mathbf{v} \bullet \mathbf{w}) \bullet \mathbf{v}$ is … (Box in one:) a scalar a vector (Neither)

b) $\|\mathbf{v}\|\mathbf{v}$ is … (Box in one:) a scalar a vector (Neither)