Math 141 Name: ________________________

MIDTERM 4 – PART 1

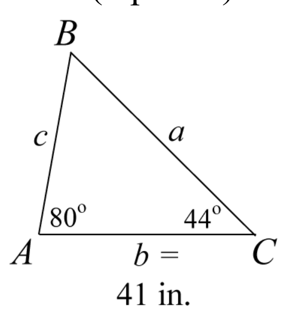
(CHAPTERS 5 AND 6: ANALYTIC & MISC. TRIGONOMETRY)
MATH 141 – SPRING 2024 – KUNIYUKI
150 POINTS TOTAL: 50 FOR PART 1, AND 100 FOR PART 2

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed.

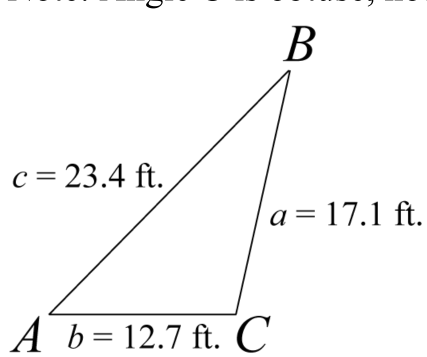
- Write units where appropriate in your final answers.
- Try to avoid rounding intermediate results; if you do round off, do it to at least five significant digits.
- We assume that all vectors on this test are in the usual xy-plane.

PART 1: USING SCIENTIFIC CALCULATORS (50 PTS.)

1) Find the length of Side c for the triangle below using the Law of Sines. Round off your answer to the nearest tenth (that is, to one decimal place) of an inch. (7 points)



2) A slanted lightning rod, represented by line segment $\overline{BC}$ in the figure below, has length 17.1 feet. An observer stands at point A. The observer's shoes are 12.7 feet from (and are level with) the base of the rod and are 23.4 feet from the top of the rod. Find the measure of Angle A, the angle of elevation from the observer's shoes to the top of the rod, using the Law of Cosines. Round off your answer to the nearest tenth (that is, to one decimal place) of a degree. Note: Angle C is obtuse, not right. (10 points)



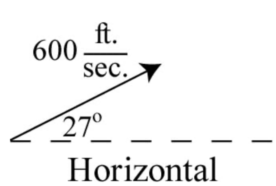
3) A vector **v** is drawn from the initial point $(4, 2)$ to the terminal point $(2, 7)$. Assume that x and y are scaled in meters. (13 points total)

a) Find the unit vector in the direction of this vector; write it in $\langle x, y \rangle$ component form. Give an **exact** answer; do **not** approximate! Rationalize denominators in your answer.

b) Find the vector of magnitude 13 meters in the direction of the vector **v**. Write the vector in $\langle x, y \rangle$ component form. Give an **exact** answer; do **not** approximate! Make sure denominators are rationalized in your answer.

c) Find $\mathbf{v} \bullet \mathbf{v}$.

4) A bullet is fired with an initial velocity of 600 feet per second, at an angle of elevation of 27° from the horizontal (see figure). Round off your answers to three significant digits and write units. (10 points total; 5 points each)

	a) What is the bullet's horizontal component of initial velocity? b) What is the bullet's vertical component of initial velocity?

5) Consider the vectors **v** and **w**, where $\mathbf{v} = \langle 4, -3 \rangle$ and $\mathbf{w} = \langle -2, 1 \rangle$. Find the angle between **v** and **w** using the formula given in class. Round off your answer to the nearest tenth (that is, to one decimal place) of a degree. Assume distance is in meters. (10 points)

Math 141 Name: ________________________

MIDTERM 4 – PART 2

(CHAPTERS 5 AND 6: ANALYTIC & MISC. TRIGONOMETRY)
MATH 141 – SPRING 2024 – KUNIYUKI
150 POINTS TOTAL: 50 FOR PART 1, AND 100 FOR PART 2

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed.

PART 2: NO CALCULATORS ALLOWED! (100 POINTS)

6) Complete the Identities. Fill out the table below so that, for each row, the left side is equivalent to the right side, based on the type of identity (ID) given in the last column. You do not have to show work. (18 points total)

Left Side	Right Side	Type of Identity (ID)
$\sin(u+v)$		Sum ID
$\cos(u+v)$		Sum ID
$\tan(u+v)$		Sum ID
$\cos(u-v)$		Difference ID
$\cos(2u)$ **Two versions**		Double-Angle ID (write any **two** of the three versions we've discussed)
$\sin^2(u)$		Power-Reducing ID (PRI)
$\cos^2(u)$		Power-Reducing ID (PRI)
$\sin\left(\frac{\theta}{2}\right)$		Half-Angle ID

7) Write the three Half-Angle Identities for $\tan\left(\frac{\theta}{2}\right)$, as given in class. (3 points)

From now on, make sure to show all steps, as in class!

8) Verify the identity: $\tan^{5/2}(x)+\tan^{1/2}(x)=\left[\sec^2(x)\right]\sqrt{\tan(x)}$. (4 points)

9) Verify the identity: $\sqrt{\dfrac{1+\cos(\theta)}{1-\cos(\theta)}}=\dfrac{1+\cos(\theta)}{\left|\sin(\theta)\right|}$.

Do **not** use $\tan\left(\dfrac{\theta}{2}\right)$ as part of your work. (7 points)

10) Use the trigonometric substitution $x = 7\tan(\theta)$ to rewrite the algebraic expression $\sqrt{x^2 + 49}$ as a trigonometric expression in θ, where θ is acute. Show all work, as in class, and simplify. (7 points)

WHEN SOLVING THE FOLLOWING TRIGONOMETRIC EQUATIONS, GIVE FINAL ANSWERS EXACTLY IN RADIANS, NOT DEGREES.

11) Find <u>all</u> real solutions of the equation: $\sec^2(x) + \sec(x) = 0$. (8 points)

12) Consider the equation: $3\tan(3x)-\sqrt{3}=0$. (15 points total)

a) Find all real solutions of the equation: $3\tan(3x)-\sqrt{3}=0$.

b) Use part a) to find the real solutions of the equation $3\tan(3x)-\sqrt{3}=0$ in the interval $[0, 2\pi)$. You do not have to use set notation, but make sure to box in all your solutions.

13) Use the Power-Reducing Identities (PRIs) to rewrite $\sin^4(x)$ using only constants, signs, and the first power of cosine expressions (and no other powers). Fill in the blanks below with real numbers in simplified form. Show all work! (13 points)

$$\sin^4(x) = \boxed{} - \boxed{}\cos(2x) + \boxed{}\cos(4x)$$

14) Find the real solutions of the equation $\cos(x) = \frac{1}{4}$ <u>in the interval</u> $[0, 2\pi)$.
Write your solution(s) as **exact** expressions; do **not** approximate! (4 points)

15) Use a Double-Angle Identity to simplify $10\sin(x)\cos(x)$. (3 points)

16) Find the simplified, exact value of: $\sin(75^\circ)\cos(15^\circ) - \cos(75^\circ)\sin(15^\circ)$.
(5 points)

17) Use **one** of the Sum-to-Product Identities below to rewrite the expression $\cos(7\theta)+\cos(3\theta)$. Simplify. (4 points)

$$\sin(x)+\sin(y)=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x)-\sin(y)=2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x)+\cos(y)=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos(x)-\cos(y)=-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

18) A triangle has one side of length a and another side of length b, and Angle C is the included angle between the two sides. Give the formula we gave in class for the area of this triangle in terms of C, a, and b. (3 points)

19) Assume that **v** and **w** are two vectors in the real plane. (6 points total)

a) $(\mathbf{v}\bullet\mathbf{w})\|\mathbf{w}\|$ is … (Box in one:) a scalar a vector (Neither)

b) $(\mathbf{v}\bullet\mathbf{w})\bullet\mathbf{v}$ is … (Box in one:) a scalar a vector (Neither)

c) $(\mathbf{v}\bullet\mathbf{w})\mathbf{w}$ is … (Box in one:) a scalar a vector (Neither)