Math 141 Name: ________________________

MIDTERM 4 – PART 1

(CHAPTERS 5 AND 6: ANALYTIC & MISC. TRIGONOMETRY)
MATH 141 – SPRING 2026 – KUNIYUKI
150 POINTS TOTAL: 47 FOR PART 1, AND 103 FOR PART 2
Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed.

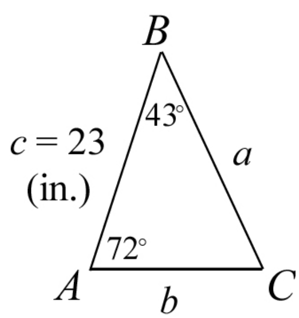
• Write units where appropriate in your final answers.

• Try to avoid rounding intermediate results; if you do round off, do it to at least five significant digits.

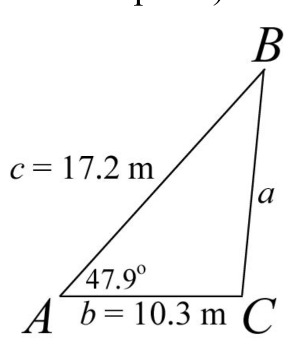
• We assume that all vectors on this test are in the usual *xy*-plane.

PART 1: USING SCIENTIFIC CALCULATORS (47 PTS.)

1) Find the length of Side b for the triangle below using the Law of Sines. Round off your answer to the nearest tenth (that is, to one decimal place) of an inch. (8 points)



2) A tilting flagpole is represented by line segment $\overline{BC}$ in the figure below. An observer stands at point A. The observer's shoes are 10.3 meters from (and are level with) the base of the pole and are 17.2 meters from the top of the pole. The angle of elevation from the observer's shoes to the top of the pole is 47.9°. Find the length of the flagpole, which is given by a, the length of $\overline{BC}$. Use the Law of Cosines. Round off your answer to the nearest tenth (that is, to one decimal place) of a meter. Note: Angle C is obtuse, not right. (7 points)



For the rest of Part 1, assume that x and y are scaled in meters.

3) For parts a) and b) below, consider the vector **v**, where $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$.
(11 points total)

a) Find the direction angle of **v**. Round off your answer to the nearest tenth (that is, to one decimal place) of a degree. (4 points)

b) Find the vector of magnitude 10 meters in the direction of the vector **v**. Write the vector in $\langle x, y \rangle$ component form. Give an **exact** answer; do **not** approximate! Rationalize denominators in your answer. (7 points)

4) Find the $\langle x, y \rangle$ component form of the vector **v** that has magnitude 4 meters and direction angle 51°. Round off the x and y components to the nearest hundredth (that is, to two decimal places) of a meter. (5 points)

5) Assume that **v** and **w** are two vectors in the real plane. (6 points total)

a) $(\mathbf{v} \bullet \mathbf{w})\|\mathbf{w}\|$ is … (Box in one:) a scalar a vector (Neither)

b) $(\mathbf{v} \bullet \mathbf{w}) \bullet \mathbf{v}$ is … (Box in one:) a scalar a vector (Neither)

c) $(\mathbf{v} \bullet \mathbf{w})\mathbf{w}$ is … (Box in one:) a scalar a vector (Neither)

6) Consider the vectors **v** and **w**, where $\mathbf{v} = \langle 4, -3 \rangle$ and $\mathbf{w} = \langle -2, 1 \rangle$. Find the angle between **v** and **w** using the formula given in class. Round off your answer to the nearest tenth (that is, to one decimal place) of a degree. (10 points)

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MIDTERM 4 – PART 2

(CHAPTERS 5 AND 6: ANALYTIC & MISC. TRIGONOMETRY)
MATH 141 – SPRING 2026 – KUNIYUKI
150 POINTS TOTAL: 47 FOR PART 1, AND 103 FOR PART 2

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed.

PART 2: NO CALCULATORS ALLOWED! (103 PTS.)

7) Complete the Identities. Fill out the table below so that, for each row, the left side is equivalent to the right side, based on the type of identity (ID) given in the last column. You do not have to show work. (16 points total)

Left Side	Right Side	Type of Identity (ID)
$\sin(u+v)$		Sum ID
$\cos(u+v)$		Sum ID
$\tan(u+v)$		Sum ID
$\sin(u-v)$		Difference ID
$\sin(2u)$		Double-Angle ID
$\sin^2(u)$		Power-Reducing ID (PRI)
$\cos^2(u)$		Power-Reducing ID (PRI)
$\cos\left(\frac{\theta}{2}\right)$		Half-Angle ID

8) Write the three versions of the Double-Angle Identities for $\cos(2u)$ that we discussed in class; these were listed on the "More Trig Identities" handout. (6 points total)

9) Write the three Half-Angle Identities for $\tan\left(\frac{\theta}{2}\right)$, as given in class. (3 points)

Make sure to show all steps, as in class!

10) Verify the identity: $\dfrac{\sin(2x)}{1+\tan^2(x)} = 2\sin(x)\cos^3(x)$. (6 points)

11) Verify the identity: $\dfrac{1+\csc(-\theta)}{\cot(\theta)} = \tan(\theta)-\sec(\theta)$. (8 points)

12) Use the trigonometric substitution $x = 7\tan(\theta)$ to rewrite the algebraic expression $\sqrt{x^2 + 49}$ as a trigonometric expression in θ, where θ is acute. Show all work, as in class, and simplify. (7 points)

WHEN SOLVING THE FOLLOWING TRIGONOMETRIC EQUATIONS, GIVE FINAL ANSWERS EXACTLY IN RADIANS, NOT DEGREES.

13) Find <u>all</u> real solutions of the equation: $\tan^2(x) + 4\tan(x) = 0$.
Give **exact** answers; do **not** approximate! (8 points)

14) Consider the equation: $2\sin(4x) - \sqrt{3} = 0$. (18 points total)

a) Find all real solutions of the equation: $2\sin(4x) - \sqrt{3} = 0$.

b) Use part a) to find the real solutions of the equation $2\sin(4x) - \sqrt{3} = 0$ in the interval $[0, 2\pi)$. You do not have to use set notation, but make sure to box in all your solutions.

15) Use **one** of the Product-to-Sum Identities below to rewrite the expression $\sin(7\theta)\cos(\theta)$. Simplify. (4 points)

$$\sin(u)\sin(v)=\frac{1}{2}\left[\cos(u-v)-\cos(u+v)\right]$$

$$\cos(u)\cos(v)=\frac{1}{2}\left[\cos(u-v)+\cos(u+v)\right]$$

$$\sin(u)\cos(v)=\frac{1}{2}\left[\sin(u+v)+\sin(u-v)\right]$$

$$\cos(u)\sin(v)=\frac{1}{2}\left[\sin(u+v)-\sin(u-v)\right]$$

16) Simplify as an exact value: $\cos(50^{\circ})\cos(20^{\circ})+\sin(50^{\circ})\sin(20^{\circ})$. (4 points)

17) Use a Double-Angle Identity to simplify $10\sin(x)\cos(x)$. (4 points)

18) Simplify: $\sin^{2}(15^{\circ})$. Hint: Use a Power-Reducing ID (PRI). Make sure your final answer is <u>not</u> a compound fraction. (5 points)

19) Use the Power-Reducing Identities (PRIs) to rewrite $\sin^4(x)$ using only constants, signs, and the first power of cosine expressions (and no other powers). Fill in the blanks below with real numbers in simplified form. Show all work! (11 points)

$$\sin^4(x) = \boxed{} - \boxed{}\cos(2x) + \boxed{}\cos(4x)$$

20) A triangle has one side of length *a* and another side of length *b*, and Angle *C* is the included angle between the two sides. Give the formula we gave in class for the area of this triangle in terms of *C*, *a*, and *b*. (3 points)