MATH 141: PRECALCULUS

MIDTERM 3: BASIC TRIG
(CHAPTER 4)

DISCLAIMER: This may or may not be a comprehensive list, but it's a very good start!
Know all aspects of these topics; I may go beyond listed subtopics.

CHAPTER 4: TRIG (INTRO)

Angles (4.1)
   Radian and Degree Measure; Conversions
   Arc Length along Circles
   Area of a Sector of a Circle
   Quadrants and Quadrantal Angles
   Classifying Angles
   Coterminal Angles ("Twin Brothers")

Trig Functions, Right Triangles, and the Unit Circle (4.2-4.4)
   Right Triangles: "SOH-CAH-TOA"
   Reciprocal, Quotient, Cofunction, Pythagorean, and Even/Odd Identities
   Right Triangles, Pythagorean Theorem, and Pythagorean Triples
   Special Triangles: $45°\text{-}45°\text{-}90°$, $30°\text{-}60°\text{-}90°$
   Unit Circle: $(\cos\theta, \sin\theta)$ as intersection point; $\tan\theta$ as slope
   THE Table: Tricks for filling it out
   Basic Trig Values in Quadrant I and Extending to Quadrants II, III, and IV:
      Use Reference Angles (in radians or degrees) and ASTC (for Signs).
      "Brothers" have the same reference angle and $\pm$ the same trig values.
         There are shortcuts for determining some families of brothers.
      Coterminal "Twins" have the same trig values.
   Finding angles with a given trig value
   Word problems

Graphs of Sine and Cosine Functions (4.5)
   Understanding graphs of $\sin\theta$ and $\cos\theta$ from the Unit Circle
   Using the Forms
$$y = a\sin[b(x-p)] + d, \quad y = a\cos[b(x-p)] + d$$
      Factoring the argument and (if $b < 0$) Even (for cos) / Odd (for sin)
         Properties.
      Interpret b.
   Domain and Range
   Comparing Graphs

Frame Method and PCAPI:

Pivot, **C**ycle Shape, **A**mplitude $= |a|$, **P**eriod $= \dfrac{2\pi}{b}$ $(b > 0)$,

Increment $= \dfrac{\text{Period}}{4}$

The "pivot" (left-center) of our "frame" is (p,d), where p is the
phase (horizontal) shift, and d is the vertical shift.
Label the grid lines (simplify as appropriate).
Draw the Cycle Shape (sin vs. cos; $a > 0$ vs. $a < 0$).
Superimpose the coordinate axes.
Be prepared to identify the five key points of the graph on the frame.

<u>Other Graphs (4.6)</u>

Understanding the graph of $\tan\theta$ (as slope) from the Unit Circle
Using the Forms
$$y = a\tan\big[b(x - p)\big] + d, \quad y = a\cot\big[b(x - p)\big] + d$$

Factoring the argument and (if $b < 0$) Odd Properties.
Domain and Range
Comparing Graphs
Frame Method and PCAPI:
Pivot (p,d) different for tan (it's at the center of the frame; it's at
the left-center, as usual, for cot)
Cycle Shapes are snakes. (tan vs. cot; $a > 0$ vs. $a < 0$)
"Amplitude" is not really amplitude, but it helps with the frame
Period $= \dfrac{\pi}{b}$ $(b > 0)$, not $\dfrac{2\pi}{b}$
Vertical Asymptotes are at the left and right edges of the frame
Label the grid lines (simplify as appropriate).
Superimpose the coordinate axes.
Be prepared to identify the three key points on the frame.
Using the Forms
$$y = a\csc\big[b(x - p)\big] + d, \quad y = a\sec\big[b(x - p)\big] + d$$

Factoring the argument and (if $b < 0$) Even (for sec) / Odd (for csc)
Properties.
Domain and Range
Even ($\sec\theta$) / Odd ($\csc\theta$) Properties
Comparing Graphs
Frame Method and PCAPI:
Draw and label sin or cos "version" first using the Frame Method.
Draw Vertical Asymptotes through midpoints (inflection points).
Draw "Up-U"s and "Down-U"s.
Period $= \dfrac{2\pi}{b}$ $(b > 0)$

<u>Inverse Trig Functions (4.7)</u>

        Restricting Domains of Trig Functions
        Graphs, Domains, and Ranges of Inverse Trig Functions
        Tricks for remembering Ranges
        Evaluating Inverse Trig Functions
        Inverse Properties (Two Groups)
        Using Right Triangles to obtain Algebraic Expressions

<u>Solving a Right Triangle and Applications (4.8)</u>

        Solving Right Triangles and Word Problems
                Draw and label figures; Define variables; Units
                Sum of Angles $= 180\degree$ or π rad
                Pythagorean Theorem
                Using Trig Functions
        Bearing and Navigation