

# MATH 141: PRECALCULUS

## **FINAL:** **SYSTEMS AND MATRICES (CHAPTERS 7 AND 8),** **DISCRETE MATH (CHAPTER 9), AND** **CONICS AND POLAR EQUATIONS (CHAPTER 10)**

DISCLAIMER: This may or may not be a comprehensive list, but it's a very good start!  
Know all aspects of these topics; I may go beyond listed subtopics.

### **CHAPTER 7: SYSTEMS**

Solving Linear and Nonlinear Systems of Equations using: (7.1, 7.2)

Substitution Method  
Addition / Elimination Method  
Graphical Method (the idea)

Some Issues: (7.1, 7.2)

Real solutions of systems correspond to intersection points.  
Consistent and Inconsistent Systems  
Systems of Linear Equations can have 0, 1, or infinitely many solutions.  
Write all coordinates of a solution in the appropriate order.

Partial Fractions and Systems (7.4)

Use Long Division when necessary  
Forms based on the factored form of the denominator  
Finding the Unknowns using "Plugging In" and "Matching Coefficients"

### **CHAPTER 8: MATRICES**

Size

Gaussian Elimination (with Back-Substitution) and Gauss-Jordan Elimination (8.1)

Steps Common to Both Methods:

Set up an Augmented Matrix  
You may want to fill in "missing" coefficients in the given system.  
Use Elementary Row Operations (EROs):  
Row Reordering  
Row Rescaling  
"Row Replacements"

For Gaussian Elimination (with Back-Substitution):

Go to Row-Echelon Form; recognize this form.  
Write the new system.  
Back-substitute.  
Write the solution set.  
Check?

For Gauss-Jordan Elimination:

Go to Reduced Row-Echelon Form (RRE Form); recognize this form.  
(Note: Required for systems with infinitely many solutions.)

When does a system have no real solutions?

Matrix Operations (8.2)

Notation

Addition and Subtraction:  $A \pm B$

Scalar Multiplication:  $cA$

Matrix Multiplication:  $AB$ ; Size

Powers:  $A^k$

Identity Matrices

Matrix Notation and Systems of Linear Equations:  $AX = B$

Matrix Inverses (8.3)

Definition; Checking Inverses

Using Gauss-Jordan Elimination to take  $[A|I]$  to  $[I|A^{-1}]$ . (General method)

Shortcut for  $2 \times 2$  Matrices

Inverse Matrix Method for Solving Systems of Linear Equations:  $X = A^{-1}B$   
(if  $A$  invertible)

Determinant of a Square Matrix (8.4)

Shortcuts for Matrices of Size  $2 \times 2$  (“Butterfly”) and  $3 \times 3$  (“Sarrus’s Rule”)

Expansion by Cofactors (General method)

Remember the Sign Matrix.

Cross Products (not on exam)

Applications of Determinants (8.5)

Cramer’s Rule for Solving Systems of Linear Equations

Geometric Applications: Area and Volume

## **CHAPTER 9: DISCRETE MATH**

### Sequences and Series (9.1, 9.2, 9.3)

#### Sequences and Related Issues

Terms  
Sign Alternators  
Even and Odd Integers  
Recursively Defined Sequences  
Factorials and Simplifications  
Arithmetic and Geometric Sequences; General  $n^{\text{th}}$ -term Formulas

#### Series

Summation (Sigma) Notation  
Convergence and Sums of Infinite Geometric Series  
Expressing repeating decimals as nice fractions using geometric series

### Mathematical Induction (9.4)

Prove using mathematical induction that:  $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2} \quad (\forall n \in \mathbf{Z}^+)$

### Binomial Theorem and Pascal's Triangle (9.5, some 9.6)

## **CHAPTER 10: CONICS AND POLAR EQUATIONS**

### Ellipses (10.3)

Standard Form for the Equation of an Ellipse  
Obtaining this Form, Completing the Square (CTS)  
“x-long” vs. “y-long” Ellipses  
Center, Foci, Vertices, Eccentricity  
Graphing

### Hyperbolas (10.4)

Standard Form for the Equation of a Hyperbola  
“x-open” vs. “y-open”  
Center, Foci, Vertices, Eccentricity, Asymptotes  
Graphing

### Graphing Polar Equations (10.8)