

**QUIZ 1A****(CHAPTER 0: PRELIMINARY TOPICS)****MATH 141 – FALL 2023 – KUNIYUKI****90 POINTS TOTAL**

**No notes or books allowed. A scientific calculator is allowed. Simplify as appropriate.**

You may assume that two-dimensional graphs are in the usual Cartesian  $xy$ -plane (distances in meters). Give exact answers, unless you are told to approximate.

**SHORTER PROBLEMS (41 POINTS)**

1) (1 point). The symbol  $\exists$  means which of the following? Box in one:

For all

There exists

Is a member of

2) (6 points total; 2 points each).

a) Write the **converse** of this given statement:

“If it is a hot day, then I want ice cream.”

b) Write the **contrapositive** of this given statement:

“If it is a hot day, then I want ice cream.”

c) Which is logically equivalent to the given statement? Box in one:

Its converse

Its inverse

Its contrapositive

3) (2 points). Mathematically express the following as an absolute value inequality: The distance between  $x$  and 2 on the real number line is less than 5.

4) (4 points). Solve the correct absolute value inequality from Problem 3); that is, solve the correct answer to Problem 3). Write the solution set in interval form (the form with parentheses and/or brackets).

5) (3 points). Simplify:  $\frac{|x-7|}{x(x-7)}$  if  $x < 7$ .

6) (2 points). Simplify  $5^{3n} \cdot 5^{4n}$ .

7) (4 points). Factor  $2x^{-6} - x^3$  over  $\mathbb{Z}$  (that is, using only integer coefficients), as in class, and rewrite the result as a fraction with no negative exponents.

8) (3 points). Simplify completely:  $\frac{x^2 - 16}{4 - x}$ .

9) (1 point). Is  $\sqrt{x^2 + 9}$  equivalent to  $x + 3$ ? Box in one:

Yes

No

10) (1 point). Is  $(3x + 6)^{1/2}$  equivalent to  $3(x + 2)^{1/2}$ , where  $x \geq -2$ ? Box in one:

Yes

No

11) (2 points). What is the equation of the  $y$ -axis in the usual  $xy$ -plane?

12) (2 points). What is the slope of any line that is perpendicular to the line

$$y = -\frac{4}{3}x + 5?$$

13) (10 points total; 2 points each). Write the formulas for the following.

Description	Formula
The <b>lateral surface area</b> of a right circular <b>cylinder</b> with base radius $r$ and height $h$	
The <b>volume</b> of a right circular <b>cylinder</b> with base radius $r$ and height $h$	
The <b>volume</b> of a right circular <b>cone</b> with base radius $r$ and height $h$	
The <b>surface area</b> of a <b>sphere</b> of radius $r$	
The <b>volume</b> of a <b>sphere</b> of radius $r$	

**LONGER PROBLEMS (49 POINTS)**

Show all work, simplify as appropriate, and use “good form and procedure” (as in class).  
Box in your final answers!

14) Simplify  $\left(\frac{2x^2x^7}{x^4}\right)^3$  completely. Your final answer must not have any negative exponents. (4 points)

15) Simplify completely:  $\frac{x^3 + 27}{2x^2 + 5x - 3}$ . (8 points)

16) (4 points). Fill in the boxes with real numbers in simplest form to make the statement correct.

$$\frac{4 - x^2}{\sqrt{x}} = 4x^{\square} - x^{\square} \quad (x > 0)$$

- 17) Simplify the following expression completely, as in class.  
Your final answer must be a single non-compound fraction with no nonpositive exponents. (You do not have to rationalize denominators.)  
You may ignore domain issues here. (9 points)

$$\frac{(2x+3)^{1/2}(12x^2) - (4x^3-1)\left(\frac{1}{2}\right)(2x+3)^{-1/2}(2)}{2x+3}$$

18) For parts a), b), and c), consider the points  $P(-2, 3)$  and  $Q(2, 8)$  in the usual  $xy$ -plane. Write all numerical constants in simplest form. Distance is measured in meters. (18 points total)

a) Find the distance between the two points (that is, the length of the line segment  $\overline{PQ}$ ). (5 points)

b) Use part a) to find the standard form of the equation of the circle centered at the point  $P(-2, 3)$  and passing through the point  $Q(2, 8)$ . (4 points)

c) Find the Slope-Intercept Form of the equation of the line  $\overline{PQ}$  that passes through the two points  $P$  and  $Q$ . Hint: You can do this part without using parts a) and b). (9 points)

- 19)  $d$  is directly proportional to the cube of  $x$  and inversely proportional to  $t$ . Find the **particular** model equation related to this statement if  $d$  is 11 when  $x$  is 2 and  $t$  is 3, as in class.

Make sure your model is in simplified form. Do not leave compound fractions in your final answer. The only real numbers allowed in the numerator and the denominator of a fraction in your final answer are integers. Any non-integer numbers must be expressed in fraction form, not decimal form.

(By “particular,” we mean determine the constant of proportionality.)  
(6 points)