

QUIZ 1A

(CHAPTER 0: PRELIMINARY TOPICS)

MATH 141 – SPRING 2023 – KUNIYUKI

90 POINTS TOTAL

No notes or books allowed. A scientific calculator is allowed. Simplify as appropriate.

You may assume that two-dimensional graphs are in the usual Cartesian xy -plane (distances in meters). Give exact answers, unless you are told to approximate.

SHORTER PROBLEMS (34 POINTS)

- 1) (1 point). The symbol \forall means which of the following? (Box in one.)
For all There exists Is a member of
- 2) (6 points total; 2 points each).
 - a) Write the **converse** of this given statement:
“If the test is easy, then I smile.”
 - b) Write the **contrapositive** of this given statement:
“If the test is easy, then I smile.”
 - c) Which is logically equivalent to the given statement? (Box in one.)
Its converse Its inverse Its contrapositive
- 3) (3 points). Give the piecewise definition of $|a|$ (where $a \in \mathbb{R}$) given in class.
- 4) (2 points). Mathematically express the following as an absolute value inequality: The distance between x and 6 on the real number line is less than 4.
- 5) (3 points). Solve the correct absolute value inequality from Problem 4); that is, solve the correct answer to Problem 4). Write the solution set in interval form (the form with parentheses and/or brackets).

6) (1 point). Is $\sqrt{a^2 + b^2}$ equivalent to $a + b$? Box in one: Yes No

7) (5 points). Factor completely over \mathbb{Z} (that is, using only integer coefficients):
 $3x^8 + 24x^5$.

8) (2 points). Fill in the boxes with simplified real numbers to make the statement correct.

$$5x^2 - \frac{8}{3}y^2 = \frac{x^2}{\boxed{}} - \frac{y^2}{\boxed{}}$$

9) (3 points). Write an equation for the line in the xy -plane that passes through the point $(3, -1)$ and that is perpendicular to the line $y = -9x + 4$. Use Point-Slope Form.

10) (8 points total; 2 points each). Write the formulas for the following.

Description	Formula
The lateral surface area of a right circular cylinder with base radius r and height h	
The volume of a right circular cylinder with base radius r and height h	
The surface area of a sphere of radius r	
The volume of a sphere of radius r	

LONGER PROBLEMS (56 POINTS)

Show all work, simplify as appropriate, and use “good form and procedure” (as in class).

Box in your final answers!

11) Factor and simplify completely $\frac{x^{-5} - 2x^{-4}}{2x - 1}$ as in class. Write your answer with no negative exponents. (4 points)

12) Simplify completely: $\frac{(7^{2n})^4}{7^n \cdot 7^{4n}}$. (4 points)

13) Fill in the boxes with simplified real numbers to make the statement correct. (4 points)

$$\frac{x^5 + 7(\sqrt[3]{x})}{7x^3} = \boxed{} x^{\boxed{}} + x^{\boxed{}} \quad (x \neq 0)$$

14) Simplify the following expression completely, as in class. Your final answer must be a single non-compound fraction with no nonpositive exponents. (You do not have to rationalize denominators.) You may ignore domain issues here. **YOU MAY USE THE BACK OF THE TEST.** (9 points)

$$\frac{x^3 \left[\frac{1}{2} (9 - x^2)^{-1/2} (-2x) \right] - (\sqrt{9 - x^2})(3x^2)}{x^6}$$

15) For parts a), b), and c), consider the points $P(3, -2)$ and $Q(1, 6)$ in the usual xy -plane. Write all numerical constants in simplest form. Distance is measured in meters. (19 points total)

a) Find the distance between the two points (that is, the length of the line segment \overline{PQ}). (5 points)

b) Find the standard form of the equation of the circle that has $P(3, -2)$ as its center and that passes through the point $Q(1, 6)$ as a solution point. Hint: Part a) will help. (5 points)

c) Find the Slope-Intercept Form of the equation of the line \overline{PQ} that passes through the two points P and Q . Hint: This part can be done without parts a) and b). (9 points)

16) A sand pile has the shape of a right circular cone with volume 46 cubic inches and height 7 inches. Find the base radius for the sand pile. Write an exact answer, as well as an approximate answer rounded off to four significant digits, and include appropriate units. (10 points)

17) Find the **particular** model equation representing the following, as in class:
“ w varies directly as x and inversely as the square of r ”
if w is 1 when x is 4 and r is 3. Make sure your model is in simplified form.
(By “particular,” we mean determine the constant of proportionality.)
(6 points)