

**QUIZ 1A****(CHAPTER 0: PRELIMINARY TOPICS)****MATH 141 – SPRING 2024 – KUNIYUKI****90 POINTS TOTAL**

**No notes or books allowed. A scientific calculator is allowed. Simplify as appropriate.**

You may assume that two-dimensional graphs are in the usual Cartesian  $xy$ -plane (distances in meters). Give exact answers, unless you are told to approximate.

**SHORTER PROBLEMS (37 POINTS)**

1) (1 point). The symbol  $\forall$  means which of the following? (Box in one.)

For all

There exists

Is a member of

2) (6 points total).

a) Write the **converse** of this given statement:

“If my candidate wins, then I celebrate.”

b) Write the **contrapositive** of this given statement:

“If my candidate wins, then I celebrate.”

c) Which is logically equivalent to the given statement? (Box in one.)

Its converse

Its inverse

Its contrapositive

3) (2 points). Mathematically express the following as an absolute value inequality: The distance between  $x$  and 5 on the real number line is less than 7.

4) (4 points). Solve the correct absolute value inequality from Problem 3); that is, solve the correct answer to Problem 3). Write the solution set in interval form (the form with parentheses and/or brackets).

5) (3 points). Give the piecewise definition of  $|a|$  (where  $a \in \mathbb{R}$ ) given in class.

6) (1 point). Write  $\sqrt[8]{x^3}$  in power form (that is,  $x$  raised to an exponent).

7) (4 points). Factor  $5x^5 + 3x^4 - 2x^3$  completely over  $\mathbb{Z}$  (that is, using only integer coefficients).

8) (4 points). Fill in the boxes with real numbers in simplest form to make the statement correct.

$$\frac{x^5 - 5\sqrt{x} + 3}{x^2} = x \boxed{\phantom{00}} - 5x \boxed{\phantom{00}} + 3x \boxed{\phantom{00}} \quad (x > 0)$$

- 9) (2 points). Fill in the boxes with simplified real numbers to make the statement correct.

$$16x^2 + \frac{25y^2}{4} = \frac{x^2}{\boxed{\phantom{000}}} + \frac{y^2}{\boxed{\phantom{000}}}$$

- 10) (2 points). What is the equation of the line in the usual  $xy$ -plane that passes through the point  $(4, 7)$  and has slope 0?
- 11) (8 points total; 2 points each). Write the formulas for the following.

Description	Formula
The <b>volume</b> of a right circular <b>cylinder</b> with base radius $r$ and height $h$	
The <b>volume</b> of a right circular <b>cone</b> with base radius $r$ and height $h$	
The <b>surface area</b> of a <b>sphere</b> of radius $r$	
The <b>volume</b> of a <b>sphere</b> of radius $r$	

**LONGER PROBLEMS (53 POINTS TOTAL)**

Show all work, simplify as appropriate, and use “good form and procedure” (as in class).  
Box in your final answers!

- 12) Simplify  $\frac{(-3x^2)^3}{x^{10}}$  completely. Your final answer must not have any negative exponents. (5 points)

13) Simplify completely:  $\frac{4-x}{x^3-64}$ . (5 points)

14) A particular right circular cylinder has lateral surface area 310 square inches and base radius 7 inches. Find the height,  $h$ , of this cylinder. Write an exact answer, and include appropriate units. Also write an approximate answer in decimal form by rounding off to four significant digits. (7 points)

15) For parts a), b), c), and d), consider the points  $P(4, -1)$  and  $Q(8, 1)$  in the usual  $xy$ -plane. Write all numerical constants in simplest form. Distance is measured in meters. (20 points total)

a) Find the distance between the two points (that is, the length of the line segment  $\overline{PQ}$ ). (5 points)

b) Use part a) to find the standard form of the equation of the circle centered at the point  $P(4, -1)$  and passing through the point  $Q(8, 1)$ . (4 points)

c) Find the Slope-Intercept Form of the equation of the line  $\overline{PQ}$  that passes through the two points  $P$  and  $Q$ . Hint: You can do this part without using parts a) and b). (9 points)

d) What is the slope of any line in the  $xy$ -plane that is perpendicular to the line  $\overline{PQ}$ ? Hint: You can do this part without using parts a) and b). Your work from part c) should help, though. (2 points)

- 16) Find the **particular** model equation representing the following, as in class:  
 $I$  is directly proportional to  $m$  and is inversely proportional to the square of  $d$ ,  
where  $I$  is 7 when  $m$  is 2 and  $d$  is 5.

Make sure your model is in simplified form. Do not leave compound fractions in your final answer. The only real numbers allowed in the numerator and the denominator of a fraction in your final answer are integers. Any non-integer numbers must be expressed in fraction form, not decimal form.

(By “particular,” we mean determine the constant of proportionality.)  
(7 points)

- 17) Simplify the following expression completely, as in class.  
Your final answer must be a single non-compound fraction with no nonpositive exponents. (You do not have to rationalize denominators.) You may ignore domain issues here. (9 points)

$$\frac{(x^2) \left[ \frac{1}{3} (3x^2 + 1)^{-2/3} (6x) \right] - \left( \sqrt[3]{3x^2 + 1} \right) (2x)}{x^4}$$