QUIZ 1A - SOLUTIONS

(CHAPTER 0: PRELIMINARY TOPICS)
MATH 141 – SPRING 2026 – KUNIYUKI
90 POINTS TOTAL

SHORTER PROBLEMS (37 POINTS)

1) (1 point). The symbol $\forall$ means which of the following? (Box in one.)

   [For all] There exists Is a member of

2) (6 points total; 2 points each).

   a) Write the **converse** of this given statement:

      "If the test is easy, then I smile."

      ["If I smile, then the test is easy."]

   b) Write the **contrapositive** of this given statement:

      "If the test is easy, then I smile."

      ["If I do not smile, then the test is not easy."]

      Note: "Not easy" is not the same thing as "hard."

   c) Which is logically equivalent to the given statement? (Box in one.)

      Its converse Its inverse **[Its contrapositive]**

3) (3 points). Give the piecewise definition of $|a|$ $\left(\text{where } a \in \mathbb{R}\right)$ given in class.

$$\boxed{|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}}$$

4) (2 points). Mathematically express the following as an absolute value inequality: The distance between x and 6 on the real number line is less than 4.

$$\boxed{|x-6| < 4, \text{ or } |6-x| < 4}$$

5) (3 points). Solve the correct absolute value inequality from Problem 4); that is, solve the correct answer to Problem 4). Write the solution set in interval form (the form with parentheses and/or brackets).

$$|x-6| < 4$$

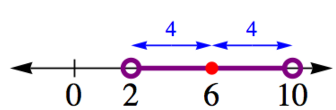
$$-4 < x - 6 < 4$$

We can add 6 to all three parts of this compound inequality.

$$2 < \quad x \quad < 10$$

The solution set in interval form is: $\boxed{(2, 10)}$.

Note: Relate this problem to #4. This is the set of numbers that are strictly within 4 units of 6 on the real number line.



6) (1 point). Is $\sqrt{a^2+b^2}$ equivalent to $a+b$? Box in one: Yes $\boxed{\text{No}}$

A counterexample is provided by the case $a=3$ and $b=4$. The square root of a sum does not typically equal the sum of the square roots. $\sqrt{a^2+b^2}$ can't be simplified nicely.

7) (5 points). Factor completely over $\mathbb{Z}$ (that is, using only integer coefficients): $3x^8+24x^5$.

First, factor out the GCF, $3x^5$: $3x^8+24x^5=3x^5\left(x^3+8\right)$.

x^3+8 is a Sum of Two Cubes, which factors as:

$A^3+B^3=\left(A+B\right)\left(A^2-AB+B^2\right)$. Here, $A=x$ and $B=2$.

The visible signs on the right-hand side follow the SOAP pattern:
"Same" (as on the left-hand side), "Opposite," and "+ (Always Plus)."

$$3x^5\left(x^3+8\right)=\boxed{3x^5\left(x+2\right)\left(x^2-2x+4\right)}$$

Note: $\left(x^2-2x+4\right)$ is irreducible (prime) over $\mathbb{Z}$ by the Test for Factorability.

8) (4 points). Fill in the boxes with real numbers in simplest form to make the statement correct.

$$\frac{4-x^2}{\sqrt{x}}=4x^{\boxed{-\frac{1}{2}}}-x^{\boxed{\frac{3}{2}}}\quad\left(x>0\right)$$

$$\frac{4-x^2}{\sqrt{x}}=\frac{4-x^2}{x^{1/2}}=\frac{4}{x^{1/2}}-\frac{x^2}{x^{1/2}}=4x^{-\frac{1}{2}}-x^{\left(2-\frac{1}{2}\right)}=4x^{-\frac{1}{2}}-x^{\frac{3}{2}}$$

9) (2 points). What is the equation of the y-axis in the usual xy-plane? $\boxed{x=0}$.

10) (2 points). What is the slope of any line that is perpendicular to the line $y=-\frac{4}{3}x+5$ in the usual xy-plane?

The slope of the given line is $-\frac{4}{3}$. The **opposite reciprocal** of $-\frac{4}{3}$ is $\frac{3}{4}$, which is the slope of any line **perpendicular** to the given line. Answer: $\boxed{\frac{3}{4}}$.

11) (8 points total; 2 points each). Write the formulas for the following.

Description	**Formula**	**Comments**
The **lateral surface area** of a right circular **cylinder** with base radius r and height h	$\text{LSA} = 2\pi rh$	Think: (Circumference of circular base) times (height)
The **volume** of a right circular **cylinder** with base radius r and height h	$V = \pi r^2 h$	Think: (Area of circular base) times (height)
The **surface area** of a **sphere** of radius r	$\text{SA} = 4\pi r^2$	
The **volume** of a **sphere** of radius r	$V = \frac{4}{3}\pi r^3$	

LONGER PROBLEMS (53 POINTS)

12) Factor and simplify completely $\frac{x^{-5} - 2x^{-4}}{2x-1}$, as in class. Write your answer with no negative exponents. (4 points)

$$\frac{x^{-5} - 2x^{-4}}{2x-1} = \frac{x^{-5}\left(1 - 2x^{-4-(-5)}\right)}{2x-1} = \frac{x^{-5}\left(1 - 2x^{1}\right)}{2x-1} = \frac{x^{-5}\overset{(-1)}{\cancel{(1-2x)}}}{\underset{(1)}{\cancel{(2x-1)}}} = -x^{-5} = \boxed{-\frac{1}{x^5}\ \left(x \neq \frac{1}{2}\right)}$$

13) Simplify $\left(\frac{2x^2x^7}{x^4}\right)^3$ completely, as in class. Write your answer with no negative exponents. (5 points)

$$\left(\frac{2x^2x^7}{x^4}\right)^3 = \left(\frac{2x^9}{x^4}\right)^3 = \left(2x^5\right)^3\ (x \neq 0) = (2)^3\left(x^5\right)^3\ (x \neq 0) = \boxed{8x^{15}\ (x \neq 0)}$$

14) Simplify the following expression completely, as in class.
Your final answer must be a single non-compound fraction with no nonpositive exponents. (You do not have to rationalize denominators.)
You may ignore domain issues here. (9 points)

$$\frac{x^3\left[\frac{1}{2}\left(9-x^2\right)^{-1/2}(-2x)\right]-\left(\sqrt{9-x^2}\right)\left(3x^2\right)}{x^6}$$

"Clean-up" simplification: (The substitution $u=\left(9-x^2\right)$ may help.)

$$=\frac{x^3\left[\frac{1}{2}u^{-1/2}(-2x)\right]-\left(\sqrt{u}\right)\left(3x^2\right)}{x^6}=\frac{x^3\left[-xu^{-1/2}\right]-\left(\sqrt{u}\right)\left(3x^2\right)}{x^6}$$

$$=\frac{-x^4u^{-1/2}-\left(3x^2\right)\left(\sqrt{u}\right)}{x^6}=\frac{\overbrace{x^2}^{(1)}\left[-x^2u^{-1/2}-3\sqrt{u}\right]}{\underbrace{x^6}_{\left(x^4\right)}}=\frac{-x^2u^{-1/2}-3\sqrt{u}}{x^4}$$

Method 1: First **factor** the numerator further.

$$=\frac{-x^2u^{-1/2}-3u^{1/2}}{x^4}=\frac{u^{-1/2}\left[-x^2-3u^{\overbrace{\frac{1}{2}-\left(-\frac{1}{2}\right)}^{=1}}\right]}{x^4}=\frac{u^{-1/2}\left[-x^2-3u\right]}{x^4}$$

$$=\frac{\left(9-x^2\right)^{-1/2}\left[-x^2-3\left(9-x^2\right)\right]}{x^4}\quad\left[\text{Sub back } u=\left(9-x^2\right).\right]$$

$$=\frac{\left(9-x^2\right)^{-1/2}\left[-x^2-27+3x^2\right]}{x^4}=\boxed{\frac{2x^2-27}{x^4\left(9-x^2\right)^{1/2}},\ \text{or}\ \frac{2x^2-27}{\left(x^4\right)\sqrt{9-x^2}}}$$

Method 2: Rewrite as a **compound (or complex) fraction**.

$$=\frac{-\frac{x^2}{\sqrt{u}}-3\sqrt{u}}{x^4}=\frac{\left[-\frac{x^2}{\sqrt{u}}-3\sqrt{u}\right]}{x^4}\cdot\frac{\sqrt{u}}{\sqrt{u}}\quad\left(\text{Distribute }\sqrt{u}\text{ to \textbf{both} terms in brackets.}\right)$$

$$=\frac{-x^2-3u}{\left(x^4\right)\sqrt{u}}\quad\left[\text{Do \textbf{not} divide powers of } u \text{ here. Sub back } u=\left(9-x^2\right).\right]$$

$$=\frac{-x^2-3\left(9-x^2\right)}{\left(x^4\right)\sqrt{9-x^2}}=\frac{-x^2-27+3x^2}{\left(x^4\right)\sqrt{9-x^2}}=\boxed{\frac{2x^2-27}{\left(x^4\right)\sqrt{9-x^2}}}$$

15) For parts a), b), and c), consider the points $P(3,-2)$ and $Q(1,6)$ in the usual *xy*-plane. Write all numerical constants in simplest form. Distance is measured in meters. (19 points total)

a) Find the distance between the two points (that is, the length of the line segment $\overline{PQ}$). (5 points)

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(1-3)^2 + (6-(-2))^2}$$
$$= \sqrt{(-2)^2 + (8)^2} = \sqrt{4+64} = \sqrt{68} \text{ or } \sqrt{4 \cdot 17} = \boxed{2\sqrt{17} \text{ meters}}$$

b) Find the standard form of the equation of the circle that has $P(3,-2)$ as its center and that passes through the point $Q(1,6)$ as a solution point. Hint: Part a) will help. (5 points)

- The center (h,k) is $P(3,-2)$.
- The radius of the circle is the distance between Point *P* and Point *Q*. We found this in part a)! We have: $r = \sqrt{68} = 2\sqrt{17}$ (meters).
- The equation of the circle:

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-3)^2 + (y-(-2))^2 = (2\sqrt{17})^2 \qquad \left(\text{Remember: } 2\sqrt{17} = \sqrt{68}.\right)$$
$$\boxed{(x-3)^2 + (y+2)^2 = 68}$$

Note: $(2\sqrt{17})^2 = (2)^2(\sqrt{17})^2 = (4)(17) = 68$, or simply observe that $(\sqrt{68})^2 = 68$.

c) Find the Slope-Intercept Form of the equation of the line $\overleftrightarrow{PQ}$ that passes through the two points *P* and *Q*. Hint: This part can be done without parts a) and b). (9 points)

Find the slope of the line: $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{6-(-2)}{1-3} = \dfrac{8}{-2} = \boxed{-4}$

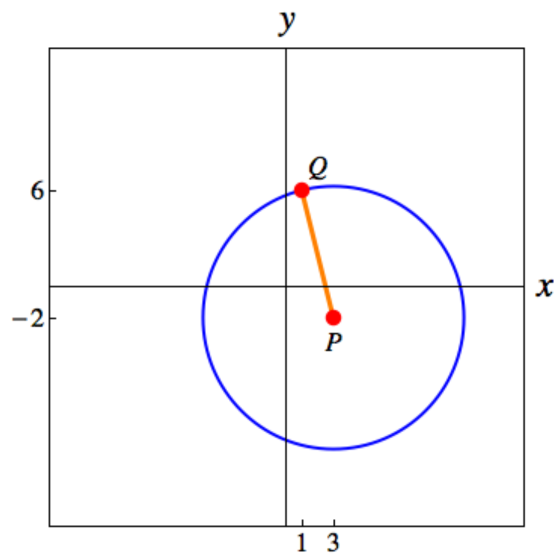
We may use either of the two given points. Let's use $Q(1,6)$.

Method 1: First use Point-Slope Form:

$$y - y_1 = m(x - x_1)$$
$$y - 6 = -4(x-1)$$
$$y - 6 = -4x + 4$$
$$\boxed{y = -4x + 10}$$

Method 2: Plug (substitute) into Slope-Intercept Form directly:

$$y = mx + b$$
$$(6) = (-4)(1) + b$$
$$b = 10 \Rightarrow \boxed{y = -4x + 10}$$



16) A sand pile has the shape of a right circular cone with volume 46 cubic inches and height 7 inches. Find the base radius for the sand pile. Write an exact answer, as well as an approximate answer rounded off to four significant digits, and include appropriate units. (10 points)

Let V = the volume of the conical sand pile.

$V = \frac{1}{3}\pi r^2 h$. Think: $\frac{1}{3}$ of the volume of the circumscribing cylinder.

Note: Cubic inches and inches are compatible units, so conversions are unnecessary.

Solve for r:

$$V = \frac{1}{3}\pi r^2 h$$
$$46 = \frac{1}{3}\pi r^2 (7)$$
$$46 = \frac{7}{3}\pi r^2$$
$$46\left(\frac{3}{7}\right)\left(\frac{1}{\pi}\right) = r^2$$
$$r^2 = \frac{138}{7\pi} \quad (r > 0)$$
$$r = \sqrt{\frac{138}{7\pi}} \text{ inches} \approx 2.505 \text{ inches}$$

$$\boxed{\text{The base radius of the sand pile is } \sqrt{\frac{138}{7\pi}} \text{ inches} \approx 2.505 \text{ inches.}}$$

Warning: If you were to approximate the radicand using a calculator, you should enter: $138 \div 7 \div \pi =$, or $138 \div (7 \times \pi) =$. According to the rules for order of operations, simply entering $138 \div 7 \times \pi =$ would give you the wrong radicand. We want a double division here.

Note: We wouldn't rationalize the denominator here, because 7π would still be irrational.

17) Find the **particular** model equation representing the following, as in class:

"*w* varies directly as *x* and inversely as the square of *r*"

if *w* is 1 when *x* is 4 and *r* is 3. Make sure your model is in simplified form. (By "particular," we mean determine the constant of proportionality.) (6 points)

General model:

$w = \frac{kx}{r^2}$, where *k* is a nonzero constant of proportionality or variation.

Find *k* using the given data:

$$1 = \frac{k(4)}{(3)^2}$$

$$1 = \frac{4k}{9}$$

$$9 = 4k$$

$$k = \frac{9}{4} \Rightarrow$$

Substitute this value of *k* into our general model to obtain our particular model:

$$w = \frac{\frac{9}{4}x}{r^2}$$

$$w = \frac{\frac{9}{4}x}{r^2} \cdot \frac{4}{4}$$

$$\boxed{w = \frac{9x}{4r^2}}$$