QUIZ 1B - SOLUTIONS

(CHAPTER 1: FUNCTIONS)
MATH 141 – SPRING 2026 – KUNIYUKI
60 POINTS TOTAL

SHORTER PROBLEMS (21 POINTS)

1) (5 points). Write the domain of f, where $f(x)=\dfrac{\sqrt[4]{x+2}}{x-3}$, using interval form, the form using parentheses and/or brackets.

- $\sqrt[4]{x+2}$ is real $\Leftrightarrow$ $x+2\geq 0$ $\Leftrightarrow$ $x\geq -2$.
- $x-3\neq 0$ $\Leftrightarrow$ $x\neq 3$, so 3 is the only exclusion from the domain based on the denominator, which is not allowed to be 0.

Here is a graph of the domain, $\text{Dom}(f)$:

In interval form, $\text{Dom}(f)=\boxed{[-2,3)\cup(3,\infty)}$.

2) (1 point). Evaluate $[\![-2.3]\!]$. (This is the same as $\lfloor -2.3\rfloor$.)

The **greatest integer** (or **floor**) function has the effect of **rounding down** arguments. $[\![-2.3]\!]=\boxed{-3}$. Note that -3 is the **greatest integer that does not exceed** -2.3.

3) (2 points). Find functions g and f such that $(f\circ g)(x)=(3x-5)^4$.
You may not use the identity function. Fill in the blanks:

$\boxed{g(x)=\underline{3x-5},\qquad f(u)=\underline{u^4}}$ (There are many other possibilities.)

4) (1 point). The graph of $y=\dfrac{1}{x^4}-x^6$ is symmetric about … (Box in one:)

the x-axis $\qquad$ $\boxed{\text{the } y\text{-axis}}$ $\qquad$ the origin $\qquad$ (none of these)

Let $f(x)=\dfrac{1}{x^4}-x^6$. $\text{Dom}(f)=\mathbb{R}\setminus\{0\}$, which is **symmetric about 0**. The function f is **even**, because $\forall x\in\mathbb{R}\setminus\{0\}$,

$$f(-x) = \frac{1}{(-x)^4}-(-x)^6 = \frac{1}{x^4}-x^6 = f(x)$$

Thus, f is an **even** function, and the graph of $y=f(x)$ is **symmetric about the *y*-axis**.

5) (1 point). The graph of $y = x^7 + 1$ is symmetric about … (Box in one:)

the x-axis   the y-axis   the origin $\boxed{\text{(none of these)}}$

Let $g(x) = x^7 + 1$. **Shortcut:** Think: $g(x) = x^7 + 1x^0$; this is a polynomial written in descending powers of x, and the exponents on x are a **mix of odd and even** integers.

$$g(-x) = (-x)^7 + 1 = -x^7 + 1$$

- g is **not odd** because it is **not** the case that $g(-x) = -g(x),\ \forall x \in \mathbb{R}$.

  Note: $\text{Dom}(g) = \mathbb{R}$, since g is polynomial.

  One counterexample is given by: $g(-1) \neq -g(1)$.

  Also, the function g is not odd because of the nonzero constant term, 1.

- g is **not even** because it is **not** the case that $g(-x) = g(x),\ \forall x \in \mathbb{R}$.

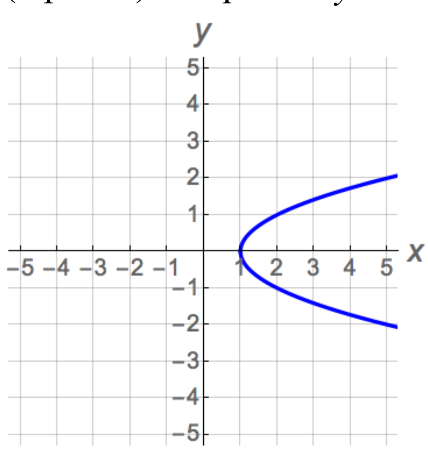
  One counterexample is given by: $g(-1) \neq g(1)$.

- Therefore, g is **neither even nor odd**, and its graph is symmetric about **neither** the y-axis nor the origin. Symmetry about the x-axis is impossible for a graph where y is a nonzero function of x.

6) (2 points). If the point $(-1, 4)$ lies on the graph of $y = f(x)$, where f is a one-to-one function, what point must then lie on the graph of $y = f^{-1}(x)$?

$\boxed{(4, -1)}$. **Reflect** $(-1, 4)$ about the line $\boldsymbol{y = x}$ by **switching** x- and y-coordinates.

7) (2 points). Graph $x = y^2 + 1$ on the grid below.



8) (1 point). Let $f(x) = \sqrt[3]{x+4}$. What is $f^{-1}(x)$?

<u>Answer</u>: $f^{-1}(x) = \boxed{x^3 - 4}$.

<u>Conceptual Approach</u>. f **adds 4** to its input and takes the **cube root** of the result.

To undo f, f^{-1} **cubes** its input and **subtracts 4** from the result; remember that inverse operations are applied in **reverse** order.

Mechanical Approach.

Step 1: Replace $f(x)$ with *y*.

$$y = \sqrt[3]{x+4}$$

Step 2: Switch *x* and *y*.

$$x = \sqrt[3]{y+4}$$

Step 3: Solve for *y*.

$$x^3 = y+4$$
$$x^3 - 4 = y$$
$$y = x^3 - 4$$

Step 4: Replace *y* with $f^{-1}(x)$.

$$f^{-1}(x) = x^3 - 4$$

9) (6 points). Match. The *x*- and *y*-axes are not necessarily scaled the same way.

The graph of $y = \sqrt[3]{x}$ is Graph **B**. (“Lazy snake” / “S” graph)

The graph of $y = x^{2/3}$ is Graph **C**. (“Bird beak/Tornado/Funnel”)

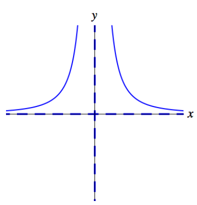
The graph of $y = \dfrac{1}{x}$ is Graph **F**. (Hyperbola)

The graph of $y = \dfrac{1}{x^2}$ is Graph **A**. (“Volcano”)

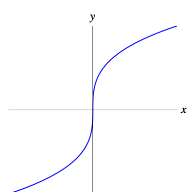
The graph of $y = |x|$ is Graph **D**. (“V” graph)

The graph of $y = \sqrt{49 - x^2}$ is Graph **E**. (Upper semicircle)

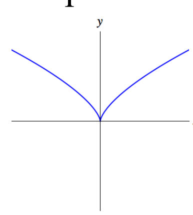
Graph A



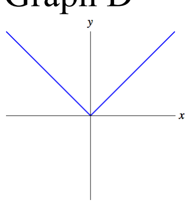
Graph B



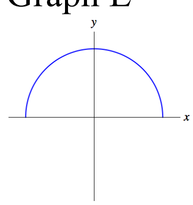
Graph C



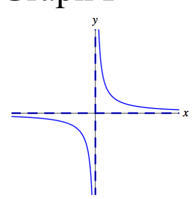
Graph D



Graph E



Graph F



- The **domains** of the functions corresponding to A, B, C, D, E, and F are, respectively, $(-\infty, 0) \cup (0, \infty)$, $\mathbb{R}$, $\mathbb{R}$, $\mathbb{R}$, $[-7, 7]$, and $(-\infty, 0) \cup (0, \infty)$.

- B and F are graphs of **odd** functions, while the others are graphs of **even** functions.

LONGER PROBLEMS (39 POINTS)

10) Find **and box in** the x-intercept[s] (if any) of the graph of $y=\sqrt{\dfrac{3x^2-8x+2}{x-5}}$ in the usual xy-plane. Hint: Factoring over $\mathbb{Z}$, the set of integers, won't work here. Think about another formula you could use. (7 points)

To find ***x*-intercepts**, we substitute ("plug in") $\mathbf{y=0}$ and solve for x. In other words, we want the **real zeros** of f, where $f(x)=\sqrt{\dfrac{3x^2-8x+2}{x-5}}$.

$$
\begin{aligned}
f(x)=0 \quad &\Leftrightarrow \quad \left(\sqrt{\frac{3x^2-8x+2}{x-5}}=0\right)\\
&\Leftrightarrow \quad \left(\frac{3x^2-8x+2}{x-5}=0\right)\\
&\Leftrightarrow \quad \left(3x^2-8x+2=0\right) \text{ and } \left(x-5\neq 0\right)\\
&\Leftrightarrow \quad \left(3x^2-8x+2=0\right) \text{ and } \left(x\neq 5\right)
\end{aligned}
$$

Find the (real) solutions of $3x^2-8x+2=0$ using the **Quadratic Formula (QF).**
We have $ax^2+bx+c=0$, where $a=3,\ b=-8,\ c=2$.

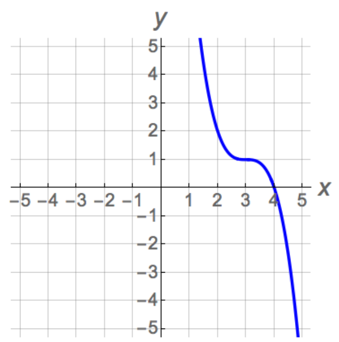
$$
\begin{aligned}
x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-(-8)\pm\sqrt{(-8)^2-4(3)(2)}}{2(3)} = \frac{8\pm\sqrt{64-24}}{6} = \frac{8\pm\sqrt{40}}{6}\\
&= \frac{8\pm\sqrt{4\cdot 10}}{6} = \frac{8\pm 2\sqrt{10}}{6} = \frac{2\left(4\pm\sqrt{10}\right)}{2(3)} = \frac{4\pm\sqrt{10}}{3}
\end{aligned}
$$

The ***x*-intercepts** are at: $\boxed{\left(\dfrac{4+\sqrt{10}}{3},\ 0\right) \text{ and } \left(\dfrac{4-\sqrt{10}}{3},\ 0\right)}$.

Note 1: Neither of our solutions is 5, so we don't have to reject either of them.

Note 2: The discriminant b^2-4ac is 40, which is **not** a perfect square, so $3x^2-8x+2$ (which had integer coefficients and whose GCF was 1) does **not** factor over the integers, $\mathbb{Z}$.

11) The graph below is obtained by taking a basic graph from Section 1.3 and applying rigid transformations. Find an equation for the graph. (5 points)



- Take the basic graph of $y = x^3$ ("rising snake").
- **Reflect** it about the ***x*-axis**; we obtain a "falling snake."

  New equation: $y = -x^3$.
- **Horizontal shift:** Translate the result 3 units to the **right**.

  New equation: $y = -(x-3)^3$.
- **Vertical shift:** Translate the result 1 unit **up**.

  Final equation: $\boxed{y = -(x-3)^3 + 1, \text{ or } y = 1 - (x-3)^3}$.

12) f is the function defined piecewise by: $f(x) = \begin{cases} x^2 + 1, & -2 \le x \le 1 \\ \sqrt{x-1}, & 1 < x < 5 \end{cases}$. (11 pts.)

a) Evaluate $f(-2)$. (1 point)

$-2 \le -2 \le 1$, so use the **top** rule:

$$f(x) = x^2 + 1 \quad \Rightarrow \quad f(-2) = (-2)^2 + 1 = 4 + 1 = \boxed{5}.$$

b) Evaluate $f(2)$. (1 point)

$1 < 2 < 5$, so use the **bottom** rule:

$$f(x) = \sqrt{x-1} \quad \Rightarrow \quad f(2) = \sqrt{(2)-1} = \sqrt{1} = \boxed{1}.$$

c) Evaluate $f(6)$. (1 point)

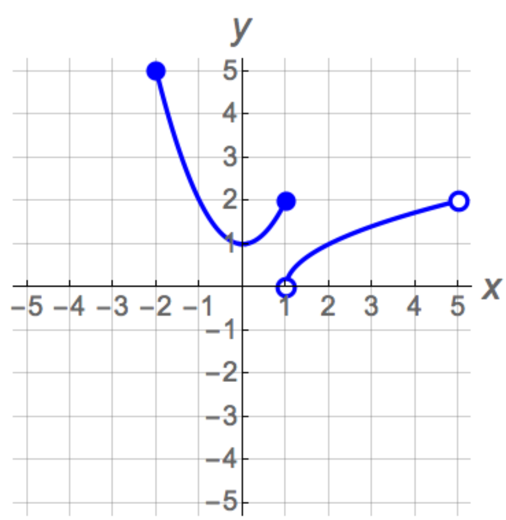
$6 \notin \text{Dom}(f)$, so $f(6)$ is $\boxed{\text{undefined}}$.

d) Graph $y = f(x)$ on the grid below. Be accurate. Clearly indicate whether endpoints are included or excluded, as in class. (6 points)

- Locate **endpoints** on the graph (corresponding to endpoints of the subdomains) and determine if they are **included** in (or **excluded** from) the graph.

Rule	**Left endpoints**	**Right endpoints**	**Shapes**
x^2+1	$x=-2$ $y = (-2)^2+1 = 5$ (see Part a).) **Left endpoint:** $(-2,5)$ **Included** because of "$\le$" (weak inequality).	$x=1$ $y = (1)^2+1 = 2$ **Right endpoint:** $(1,2)$ **Included** because of "$\le$" (weak inequality).	The **parabola** with equation $y=x^2+1$ is the graph of $y=x^2$ shifted one unit **up**. We take a piece of this parabola.
$\sqrt{x-1}$	$x=1$ $y = \sqrt{(1)-1} = \sqrt{0} = 0$ **Left endpoint:** $(1,0)$ **Excluded** because of "<" (strict inequality).	$x=5$ $y = \sqrt{(5)-1} = \sqrt{4} = 2$ **Right endpoint:** $(5,2)$ **Excluded** because of "<" (strict inequality).	The **"hook"** with equation $y=\sqrt{x-1}$ is the graph of $y=\sqrt{x}$ shifted one unit **to the right**. We take a piece of this "hook."

From Part b), we see that $f(2)=1$, so the point $(2,1)$ is also on the graph.



e) Give the **domain** of the function f using interval form (the form with parentheses and/or brackets). (1 point)

Dom$(f) = \boxed{[-2,5)}$, the **union** of the indicated subdomains: $[-2,1]\cup(1,5)$. The domain corresponds to the ***x*-coordinates** "picked up" by the graph of $y=f(x)$.

f) Give the **range** of the function f using interval form (the form with parentheses and/or brackets). (1 point)

Range$(f) = \boxed{(0,5]}$. The range corresponds to the ***y*-coordinates** "picked up" by the graph of $y=f(x)$.

13) If $f(x) = 4x^3 - x$, find the average rate of change of f from $x = 2$ to $x = 6$. Assume that x is length measured in inches and $f(x)$ is cost measured in dollars. Write the appropriate unit in your final answer. (7 points)

f is a polynomial function, so the usual formula $\dfrac{f(b) - f(a)}{b - a}$ applies.

$$\frac{f(6) - f(2)}{6 - 2} = \frac{\left[4(6)^3 - (6)\right] - \left[4(2)^3 - (2)\right]}{4} = \frac{858 - 30}{4} = \frac{828}{4} = \boxed{207 \frac{\$}{\text{in}}}$$

14) Let $f(x) = \dfrac{3}{x}$. Simplify the difference quotient completely: (9 points)

$$\frac{f(x+h) - f(x)}{h}$$

Show all work!

- When you see $f(x+h)$, think "**substitution**."

$$\frac{f(x+h) - f(x)}{h} = \frac{\dfrac{3}{x+h} - \dfrac{3}{x}}{h}$$

Multiply the numerator and the denominator by the LCD, $x(x+h)$.

$$= \frac{x(x+h)\left(\dfrac{3}{x+h} - \dfrac{3}{x}\right)}{x(x+h)(h)} = \frac{x\cancel{(x+h)}\left(\dfrac{3}{\cancel{x+h}}\right) - \cancel{x}(x+h)\left(\dfrac{3}{\cancel{x}}\right)}{xh(x+h)}$$

$$= \frac{3x - 3(x+h)}{xh(x+h)} = \frac{\cancel{3x} - \cancel{3x} - 3h}{xh(x+h)} = \frac{-3\overset{(1)}{\cancel{h}}}{x\underset{(1)}{\cancel{h}}(x+h)} = \boxed{-\frac{3}{x(x+h)} \text{ or } -\frac{3}{x^2 + xh} \quad (h \neq 0)}$$

- As h approaches 0, this approaches $-\dfrac{3}{x^2}$, which is $f'(x)$, the **derivative** of $f(x)$.

Alternate method:

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{3}{x+h} - \frac{3}{x}}{h} \quad \text{(Make the numerator a single fraction.)}$$

$$= \frac{\frac{3}{(x+h)}\cdot\frac{x}{x} - \frac{3}{x}\cdot\frac{(x+h)}{(x+h)}}{h} = \frac{\frac{3x}{x(x+h)} - \frac{3(x+h)}{x(x+h)}}{h} = \frac{\frac{3x-3(x+h)}{x(x+h)}}{h}$$

$$= \frac{\frac{\cancel{3x-3x}-3h}{x(x+h)}}{h} = \frac{-3\overset{(1)}{\cancel{h}}}{x(x+h)}\cdot\frac{1}{\underset{(1)}{\cancel{h}}} = \boxed{-\frac{3}{x(x+h)} \text{ or } -\frac{3}{x^2+xh} \quad (h \neq 0)}$$