Math 150 Name: ________________________

FINAL - PART 1

MATH 150 – FALL 2016 – KUNIYUKI

PART 1: 135 POINTS, PART 2: 115 POINTS, TOTAL: 250 POINTS

No notes, books, or calculators allowed.

135 points: 45 problems, 3 pts. each. You do not have to algebraically simplify or box in your answers, unless you are instructed to. Fill in all blanks after "=" signs.

DERIVATIVES (66 POINTS TOTAL)

$D_x\left(x^{\sqrt{2}}\right) =$

$D_x\left[x^3\cos(x)\right] =$

$D_x\left(\dfrac{x^7}{2x-5}\right) =$

(Use the Quotient Rule!)

$D_x\left[\left(e^x+4\right)^6\right] =$

$D_x\left[\tan(x)\right] =$

$D_x\left[\cot(x)\right] =$

$D_x\left[\sec(x)\right] =$

$D_x\left[\csc(x)\right] =$

$D_x\left[\sin(4x+7)\right] =$

$D_x\left(e^{\frac{1}{3}x}\right) =$

TURN OVER THE SHEET! THERE'S MORE!

MORE!

$D_x\left(5^x\right)=$

$D_x\left(10^{x^3}\right)=$

$D_x\left[\ln\left(7x^2+1\right)\right]=$

$D_x\left[\log_9(x)\right]=$

$D_x\left[\sin^{-1}(x)\right]=$

$D_x\left[\cos^{-1}(x)\right]=$

$D_x\left[\tan^{-1}(x)\right]=$

$D_x\left[\sec^{-1}(x)\right]=$ (Assume the usual range for $\sec^{-1}(x)$ in our class.)

$D_x\left[\tan^{-1}(7x)\right]=$

$D_x\left[\sinh(x)\right]=$

$D_x\left[\cosh(x)\right]=$

$D_x\left[\operatorname{sech}(x)\right]=$

INDEFINITE INTEGRALS (42 POINTS TOTAL)

$\int x^9 \, dx =$

$\int \frac{1}{x} \, dx =$

$\int e^{-7x} \, dx =$

$\int 6^x \, dx =$

$\int \sin(x) \, dx =$

$\int \tan(x) \, dx =$

$\int \cot(x) \, dx =$

$\int \sec(x) \, dx =$

$\int \csc(x) \, dx =$

$\int \cos(3x) \, dx =$

$\int \sec^2(x) \, dx =$

$\int \frac{1}{36 + x^2} \, dx =$

$\int \frac{1}{\sqrt{36 - x^2}} \, dx =$

$\int \cosh(x) \, dx =$

<ins>WARNING</ins>: YOU'VE BEEN DEALING WITH INDEFINITE INTEGRALS. DID YOU FORGET SOMETHING?

TURN OVER THE SHEET! THERE’S MORE!

INVERSE TRIGONOMETRIC FUNCTIONS (6 POINTS TOTAL)

- $\lim_{x \to \infty} \tan^{-1}(x) =$ (Drawing a graph may help.)

- If $f(x) = \sin^{-1}(x)$, what is the range of f in interval form (the form with parentheses and/or brackets)? $\text{Range}(f) =$

HYPERBOLIC FUNCTIONS (6 POINTS TOTAL)

- The definition of $\sinh(x)$ (as given in class) is: $\sinh(x) =$

- Complete the following identity: $\cosh^2(x) - \sinh^2(x) =$
  (We mentioned this identity in class.)

TRIGONOMETRIC IDENTITIES (15 POINTS TOTAL)

Complete each of the following identities, based on the type of identity given.

- $\tan^2(x) + 1 =$ (Pythagorean Identity)

- $\cos(-x) =$ (Even/Odd Identity)

- $\sin(2x) =$ (Double-Angle Identity)

- $\cos(2x) =$ (Double-Angle Identity)
  (For $\cos(2x)$, I gave you three versions; you may pick any one.)

- $\sin^2(x) =$ (Power-Reducing Identity)

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FINAL - PART 2

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Show all work, simplify as appropriate, and use "good form and procedure" (as in class). Box in your final answers! You may not use L'Hôpital's Rule for finding limits. A scientific calculator and an appropriate sheet of notes are allowed on this final part.

USE THE BACK OF THIS TEST IF YOU NEED MORE ROOM!

1) Find the following limits. Each answer will be a real number, ∞, $-\infty$, or DNE (Does Not Exist). Write ∞ or $-\infty$ when appropriate. If a limit does not exist, and ∞ and $-\infty$ are inappropriate, write "DNE." **Box in your final answers.** (14 points total)

a) $\displaystyle\lim_{x\to\infty}\frac{\cos\left(x^3\right)}{x^4}$ **Show all work, as in class.** (6 points)

b) $\displaystyle\lim_{x\to-2^+}\frac{x}{x^2-3x-10}$ **Show all work, as in class.** (6 points)

c) $\displaystyle\lim_{r\to\infty}\frac{r^3+1}{\left(r^5+r\right)^2}$ **Answer only is fine.** (2 points)

2) Let $f(x)=\begin{cases}x+4, & x\neq 3\\ 9, & x=3\end{cases}$. Classify the discontinuity at $x=3$. Box in one: (2 points)

Infinite discontinuity Jump discontinuity Removable discontinuity

3) Use the limit definition of the derivative to prove that $D_x\left(5x^2 - x + 7\right) = 10x - 1$, $\forall x \in \mathbb{R}$. Do **not** use derivative short cuts we have used in class. (10 points)

4) Let $f(x) = \log_4(x)$. Consider the graph of $y = f(x)$ in the usual *xy*-plane. Find a Point-Slope Form of the **equation** of the tangent line to the graph at the point where $x = 16$. Give exact values; you do <u>not</u> have to approximate. (8 points)

5) Assume that x and y are differentiable functions of t. Evaluate $D_t\left(e^{xy}\right)$ when $x=3$, $y=5$, $\dfrac{dx}{dt}=2$, and $\dfrac{dy}{dt}=-4$. (8 points)

6) Let $f\left(x\right)=x^3-2x^2-4x+1$. (14 points total)

a) Find the two critical numbers of f.

b) Consider the graph of $y=f\left(x\right)$, although you do not have to draw it. Use the First Derivative Test to classify the point at $x=2$ as a local maximum point, a local minimum point, or neither.

c) Use the Second Derivative Test to classify the point at the **other** critical number as a local maximum point or a local minimum point.

7) Evaluate the following integrals. (20 points total)

a) $\int_{9}^{16} \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$. Give an exact answer. (10 points)

b) $\int \frac{7x}{x^4+81}\,dx$ (10 points)

Hint: Consider the Chapter 8 material on inverse trigonometric functions!

8) The velocity function for a particle moving along a coordinate line $\left(\text{for } t > 0\right)$ is given by $v\left(t\right) = \frac{1}{t^4} - \sqrt{t}$, where t is time measured in seconds and velocity is given in meters per second. The particle's position is measured in meters. Find $s\left(t\right)$, the corresponding position function [rule], if $s\left(1\right) = 2$ (meters). (9 points)

9) The region R is bounded by the x-axis, the y-axis, and the graphs of $y = \cos(x)$ and $x = \dfrac{\pi}{4}$ in the usual xy-plane. **Sketch and shade in the region R.** Find the **volume** of the solid generated by revolving R about the x-axis. **Evaluate** your integral completely. Give an **exact** answer in simplest form with appropriate units. Distances and lengths are measured in meters. Hint: Use a Power-Reducing Identity. (18 points)

10) Rewrite $\tan\left(\sin^{-1}\left(\frac{x}{5}\right)\right)$ as an algebraic expression in x, where $0 < x < 5$.

(7 points)

11) Find $D_r\left[\sin^{-1}\left(\sinh\left(r\right)\right)\right]$. (5 points)