MATH 150: OUTLINE FOR THE FINAL

FUNCTIONS and TRIGONOMETRY REVIEW (CHAPTER 1)

- Finding the (implied) domain of a function
- Even / odd functions and symmetry of their graphs
- Composite functions
- Finding exact trigonometric values
- Trigonometric functions and identities (Memorize and use Fundamental and Advanced Identities; use Product-to-Sum and Sum-to-Product Identities)
- Domains, ranges, and graphs of the six basic trigonometric functions
- Simplifying trigonometric expressions and verifying trigonometric identities
- Solving trigonometric equations

LIMITS AND CONTINUITY (CHAPTER 2)

- Finding limits using basic properties and such tools as simple evaluation; sign analysis; limit forms; “long-run” limits involving \( \lim_{x \to \infty} \) or \( \lim_{x \to -\infty} \) : dividing a numerator and a denominator by the highest power of (say) \( x \) in the denominator, short cuts for rational functions, dominant term substitution (“DTS”) short cuts
- One-sided vs. two-sided limits
- Recognizing when a limit does not exist (DNE); \( \infty \) and \( -\infty \) are special cases
- Horizontal asymptotes (“HAs”) and “long-run” limits; know short cuts
- Vertical asymptotes (“VAs”) and infinite limits at a point; sign analysis; know short cuts
- Indeterminate forms, especially Limit Form 0/0:
  - Tools include factoring and canceling/dividing, rationalizing; VAs vs. holes
  - Squeeze (Sandwich) Theorem: applying it in “local” and “long-run” cases
- Rigorous \( \varepsilon-\delta \) definition of \( \lim_{x \to a} f(x) = L \)
- Continuity
  - Defining it at a point, on an open interval, [recognize it] on other interval types
  - Classifying discontinuities: removable, jump, infinite
  - Where is \( f \) continuous / discontinuous?
  - The Intermediate Value Theorem (IVT) and the idea of the Bisection Method for approximating a zero (root) of \( f \)

DERIVATIVES (CHAPTER 3)

- Rectilinear motion and projectile problems
  - position \( s(t) \), velocity \( v(t) \), acceleration \( a(t) \); units
- The limit definition of the derivative and using it to find derivatives
- Tangent lines, normal lines, and their equations; derivatives as slopes of tangent lines
- Average rate of change on an interval vs. instantaneous rate of change at a point
- Basic differentiation rules such as the Linearity, Product, Quotient, Power, and Chain Rules (leading to Generalized Power and Trigonometric Rules)
- Notation for derivatives, including higher-order derivatives
- Where is \( f \) differentiable / not differentiable? Also: corners, cusps, vertical tangent lines
- Finding \( D_a(\sin x) \) and \( D_a(\cos x) \) using the limit def’n of derivative \( \leftarrow \) not on the Final
- Finding derivatives of other trigonometric functions using the Quotient or Reciprocal rule
- Differentials and linearization of functions
- Implicit differentiation (“Imp Diff”)
- Related rates

  If you get word problems on the Final, they will not involve elaborate setups.
APPLICATIONS OF DERIVATIVES (CHAPTER 4)

Finding critical numbers ("CNs") & corresponding points, "PINs," inflection points (IPs)

The Extreme Value Theorem (EVT) and finding absolute maximum and minimum points
for the graph of a function on a closed interval

Rolle’s Theorem and the Mean Value Theorem (MVT) for Derivatives

Using the First and Second Derivative Tests to classify points at critical numbers (CNs)
as local maximum points, local minimum points, or neither.

Using the first derivative to see where a function $f$ is increasing vs. decreasing
Using the second derivative to see where the graph of $f$ is concave up vs. concave down

Optimization problems (see my comment under Related rates)

Rectilinear motion and projectile problems

Newton’s Method for approximating a zero (root) of $f$

INTEGRALS (CHAPTER 5)

Indefinite integrals (write “ + C”) vs. definite integrals (think “sum of signed areas”)

Solving differential equations subject to initial conditions, including physical applications

Basic rules: Linearity, Power, Trigonometric

$u$-substitutions

When evaluating definite integrals: Change the limits of integration immediately,
or work out the corresponding indefinite integral first and then apply the FTC

Using geometry to evaluate definite integrals

Defining a definite integral as a limit of Riemann sums

Properties of integrals

The Mean Value Theorem (MVT) for Integrals, $f_{av}$: average value of $f$ on an interval

The Fundamental Theorem of Calculus (FTC), parts I and II

Part II is essential for evaluating definite integrals

Numerical approximation of definite integrals

Left-hand, Right-hand, and Midpoint Riemann Approximations using rectangles

Trapezoidal Rule and Simpson’s Rule $\leftarrow$ (formulas would be given)

APPLICATIONS OF INTEGRALS (CHAPTER 6)

Finding areas (Tools include: Solving an equation for a variable, finding intersection
points, finding which graph is on the top / bottom / right / left of a region, etc.)

Finding volumes by using cross sections, including…

Finding volumes of solids of revolution using the Disk / Washer Method

Finding volumes of solids of revolution using the Cylinder (Cylindrical Shells) Method

Arc length and surface areas of surfaces of revolution

LOGARITHMIC and EXPONENTIAL FUNCTIONS (CHAPTER 7)

Defining $\ln x$

Differentiation and integration

Logarithmic differentiation ("Log Diff") and laws of logarithms

Integrating $\tan x, \cot x, \sec x, \csc x$: proofs and results

Working with $e$ and bases other than $e$; Change of Base Property

INVERSE TRIGONOMETRIC and HYPERBOLIC FUNCTIONS (CHAPTER 8)

Evaluating and differentiating inverse trigonometric functions;
integrals yielding inverse trigonometric results can require unusual $u$-substitutions

Graphs, domains, and ranges of the three basic inverse trigonometric functions

Using right triangles when combining trigonometric and inverse trigonometric functions

Hyperbolic functions: definitions, evaluations, identities, derivatives, integrals