# MATH 150: OUTLINE FOR THE FINAL

# **FUNCTIONS and TRIGONOMETRY REVIEW (CHAPTER 1)**

Finding the (implied) domain of a function

Even / odd functions and symmetry of their graphs

Composite functions

Finding exact trigonometric values

Trigonometric functions and identities (Memorize and use Fundamental and Advanced Identities; use Product-to-Sum and Sum-to-Product Identities)

Domains, ranges, and graphs of the six basic trigonometric functions

Simplifying trigonometric expressions and verifying trigonometric identities

Solving trigonometric equations

# **LIMITS AND CONTINUITY (CHAPTER 2)**

Finding limits using basic properties and such tools as simple evaluation; sign analysis; limit forms; "long-run" limits involving  $\lim_{x\to\infty}$  or  $\lim_{x\to-\infty}$ : dividing a numerator

and a denominator by the highest power of (say) *x* in the denominator, short cuts for rational functions, dominant term substitution ("DTS") short cuts

One-sided vs. two-sided limits

Recognizing when a limit does not exist (DNE);  $\infty$  and  $-\infty$  are special cases

Horizontal asymptotes ("HAs") and "long-run" limits; know short cuts

Vertical asymptotes ("VAs") and infinite limits at a point; sign analysis; know short cuts Indeterminate forms, especially Limit Form 0/0:

Tools include factoring and canceling/dividing, rationalizing; VAs vs. holes Squeeze (Sandwich) Theorem: applying it in "local" and "long-run" cases

Rigorous  $\varepsilon$ - $\delta$  definition of  $\lim_{x \to a} f(x) = L$ 

Continuity

Defining it at a point, on an open interval, [recognize it] on other interval types Classifying discontinuities: removable, jump, infinite

Where is f continuous / discontinuous?

The Intermediate Value Theorem (IVT) and the idea of the Bisection Method for approximating a zero (root) of f

## **DERIVATIVES (CHAPTER 3)**

Rectilinear motion and projectile problems

position s(t), velocity v(t), acceleration a(t); units

The limit definition of the derivative and using it to find derivatives

Tangent lines, normal lines, and their equations; derivatives as slopes of tangent lines

Average rate of change on an interval vs. instantaneous rate of change at a point

Basic differentiation rules such as the Linearity, Product, Quotient, Power, and

Chain Rules (leading to Generalized Power and Trigonometric Rules)

Notation for derivatives, including higher-order derivatives

Where is f differentiable / not differentiable? Also: corners, cusps, vertical tangent lines

Finding  $D_{x}(\sin x)$  and  $D_{x}(\cos x)$  using the limit def'n of derivative  $\leftarrow$  not on the Final

Finding derivatives of other trigonometric functions using the Quotient or Reciprocal rule Differentials and linearization of functions

Implicit differentiation ("Imp Diff")

Related rates

If you get word problems on the Final, they will not involve elaborate setups.

#### **APPLICATIONS OF DERIVATIVES (CHAPTER 4)**

Finding critical numbers ("CNs") & corresponding points, "PINs," inflection points (IPs) The Extreme Value Theorem (EVT) and finding absolute maximum and minimum points for the graph of a function on a closed interval

Rolle's Theorem and the Mean Value Theorem (MVT) for Derivatives

Using the First and Second Derivative Tests to classify points at critical numbers (CNs) as local maximum points, local minimum points, or neither.

Using the first derivative to see where a function f is increasing vs. decreasing

Using the second derivative to see where the graph of f is concave up vs. concave down

Optimization problems (see my comment under Related rates)

Rectilinear motion and projectile problems

Newton's Method for approximating a zero (root) of f

# **INTEGRALS (CHAPTER 5)**

Indefinite integrals (write "+C") vs. definite integrals (think "sum of signed areas") Solving differential equations subject to initial conditions, including physical applications Basic rules: Linearity, Power, Trigonometric u-substitutions

When evaluating definite integrals: Change the limits of integration immediately, or work out the corresponding indefinite integral first and then apply the FTC

Using geometry to evaluate definite integrals

Defining a definite integral as a limit of Riemann sums

Properties of integrals

The Mean Value Theorem (MVT) for Integrals,  $f_{av}$ : average value of f on an interval

The Fundamental Theorem of Calculus (FTC), parts I and II

Part II is essential for evaluating definite integrals

Numerical approximation of definite integrals

Left-hand, Right-hand, and Midpoint Riemann Approximations using rectangles Trapezoidal Rule and Simpson's Rule ← (formulas would be given)

## **APPLICATIONS OF INTEGRALS (CHAPTER 6)**

Finding areas (Tools include: Solving an equation for a variable, finding intersection points, finding which graph is on the top / bottom / right / left of a region, etc.) Finding volumes by using cross sections, including...

Finding volumes of solids of revolution using the Disk / Washer Method Finding volumes of solids of revolution using the Cylinder (Cylindrical Shells) Method Arc length and surface areas of surfaces of revolution

# **LOGARITHMIC and EXPONENTIAL FUNCTIONS (CHAPTER 7)**

Defining  $\ln x$ 

Differentiation and integration

Logarithmic differentiation ("Log Diff") and laws of logarithms

Integrating  $\tan x$ ,  $\cot x$ ,  $\sec x$ ,  $\csc x$ : proofs and results

Working with e and bases other than e; Change of Base Property

## **INVERSE TRIGONOMETRIC and HYPERBOLIC FUNCTIONS (CHAPTER 8)**

Evaluating and differentiating inverse trigonometric functions;

integrals yielding inverse trigonometric results can require unusual *u*-substitutions Graphs, domains, and ranges of the three basic inverse trigonometric functions Using right triangles when combining trigonometric and inverse trigonometric functions Hyperbolic functions: definitions, evaluations, identities, derivatives, integrals