Math 150 Name: ________________________

FINAL - PART 1

MATH 150 – SPRING 2017 – KUNIYUKI
PART 1: 135 POINTS, PART 2: 115 POINTS, TOTAL: 250 POINTS

No notes, books, or calculators allowed.

135 points: 45 problems, 3 pts. each. You do not have to algebraically simplify or box in your answers, unless you are instructed to. Fill in all blanks after "=" signs.

DERIVATIVES (66 POINTS TOTAL)

$D_x\left(x^e\right)=$

$D_x\left[x^4\sin(x)\right]=$

$D_x\left(\dfrac{x^3}{2x^5+8}\right)=$

(Use the Quotient Rule!)

$D_x\left(\left[\ln(x)+1\right]^4\right)=$

$D_x\left[\tan(x)\right]=$

$D_x\left[\cot(x)\right]=$

$D_x\left[\sec(x)\right]=$

$D_x\left[\csc(x)\right]=$

$D_x\left[\cos(3x-4)\right]=$

$D_x\left(e^{-7x}\right)=$

TURN OVER THE SHEET! THERE'S MORE!

MORE!

$D_x\left(9^x\right)=$

$D_x\left(7^{x^4+x}\right)=$

$D_x\left[\ln\left(6x+1\right)\right]=$

$D_x\left[\log_3\left(x\right)\right]=$

$D_x\left[\sin^{-1}\left(x\right)\right]=$

$D_x\left[\cos^{-1}\left(x\right)\right]=$

$D_x\left[\tan^{-1}\left(x\right)\right]=$

$D_x\left[\sec^{-1}\left(x\right)\right]=$ (Assume the usual range for $\sec^{-1}\left(x\right)$ in our class.)

$D_x\left[\tan^{-1}\left(e^x\right)\right]=$

$D_x\left[\sinh\left(x\right)\right]=$

$D_x\left[\cosh\left(x\right)\right]=$

$D_x\left[\operatorname{sech}\left(x\right)\right]=$

INDEFINITE INTEGRALS (42 POINTS TOTAL)

$\int x^5 \, dx =$

$\int \frac{1}{x} \, dx =$

$\int e^{3x} \, dx =$

$\int 8^x \, dx =$

$\int \cos(x) \, dx =$

$\int \tan(x) \, dx =$

$\int \cot(x) \, dx =$

$\int \sec(x) \, dx =$

$\int \csc(x) \, dx =$

$\int \sin(7x) \, dx =$

$\int \csc(x)\cot(x) \, dx =$

$\int \frac{1}{\sqrt{49 - x^2}} \, dx =$

$\int \frac{1}{49 + x^2} \, dx =$

$\int \cosh(x) \, dx =$

WARNING: YOU'VE BEEN DEALING WITH INDEFINITE INTEGRALS.
DID YOU FORGET SOMETHING?

TURN OVER THE SHEET! THERE'S MORE!

INVERSE TRIGONOMETRIC FUNCTIONS (6 POINTS TOTAL)

• If $f(x) = \cos^{-1}(x)$, what is the range of f in interval form (the form with parentheses and/or brackets)? $\text{Range}(f) =$

• $\lim\limits_{x \to 1^-} \sin^{-1}(x) =$ (Drawing a graph may help.)

HYPERBOLIC FUNCTIONS (6 POINTS TOTAL)

• The definition of $\cosh(x)$ (as given in class) is: $\cosh(x) =$

• Complete the following identity: $\cosh^2(x) - \sinh^2(x) =$
(We mentioned this identity in class.)

TRIGONOMETRIC IDENTITIES (15 POINTS TOTAL)

Complete each of the following identities, based on the type of identity given.

- $1 + \cot^2(x) =$ (Pythagorean Identity)
- $\sin(-x) =$ (Even/Odd Identity)
- $\sin(2x) =$ (Double-Angle Identity)
- $\cos(2x) =$ (Double-Angle Identity)
  (For $\cos(2x)$, I gave you three versions; you may pick any one.)
- $\cos^2(x) =$ (Power-Reducing Identity)

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FINAL - PART 2

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PART 1: 135 POINTS, PART 2: 115 POINTS, TOTAL: 250 POINTS

Show all work, simplify as appropriate, and use "good form and procedure" (as in class). Box in your final answers! You may not use L'Hôpital's Rule for finding limits. A scientific calculator and an appropriate sheet of notes are allowed on this final part.

1) Find the following limits. Each answer will be a real number, ∞, $-\infty$, or DNE (Does Not Exist). Write ∞ or $-\infty$ when appropriate. If a limit does not exist, and ∞ and $-\infty$ are inappropriate, write "DNE." **Box in your final answers.** (16 points total)

a) $\lim\limits_{r \to \infty} \dfrac{11r^4 - 7}{8r^4 + r^2 - 1}$ **Answer only is fine.** (2 points)

b) $\lim\limits_{t \to -\infty} \dfrac{t^5 + 3t^2 - 7}{t^6 - t - 1}$ **Answer only is fine.** (2 points)

c) $\lim\limits_{x \to 5^-} \dfrac{2x + 1}{x^2 - 2x - 15}$ **Show all work, as in class.** (6 points)

d) $\lim\limits_{x \to 0} \left[x^2 \cos\left(\dfrac{1}{x^3} \right) \right]$ **Show all work, as in class.** (6 points)

2) Use the limit definition of the derivative to prove that $D_x\left(\frac{1}{x}\right) = -\frac{1}{x^2}$ for all real $x \neq 0$. Do **not** use derivative short cuts we have used in class. (11 points)

3) Consider the given equation $4y^2 + 3x^4 y + 5e^y = 22 + 5e^2$. Assume that it "determines" an implicit differentiable function f such that $y = f(x)$.

Find $\frac{dy}{dx}$ (you may use the y' notation, instead). (12 points)

4) Consider the graph of the equation in Problem 3), $4y^2 + 3x^4 y + 5e^y = 22 + 5e^2$, in the usual *xy*-plane. Find a Point-Slope Form for the equation of the tangent line to the graph at the point $(-1, 2)$. Give an exact answer; do not approximate. You may use your work from Problem 3). (7 points)

5) Let $f(x) = x^3 + 6x^2 - 7$. Show all work, as in class. (17 points total)

a) Give the x-interval(s) on which f is increasing. Write your answer in interval form. Do not worry about the issue of parentheses vs. brackets.

b) Give the x-interval(s) on which the graph of $y = f(x)$ is concave up. Write your answer in interval form. Do not worry about the issue of parentheses vs. brackets.

6) Evaluate the following integrals. (17 points total)

a) $\int \sin^2(\theta)\, d\theta$ (7 points)

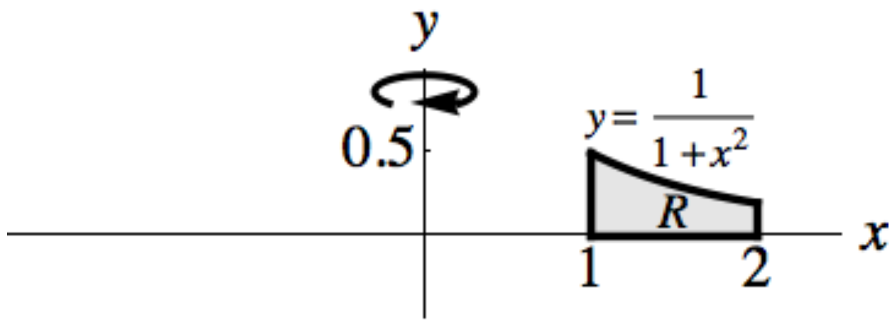
b) $\int \frac{x}{\sqrt{25-9x^4}}\,dx$ (10 points)

Hint: Consider the Chapter 8 material on inverse trigonometric functions!

7) Rewrite $\cos\left(\tan^{-1}\left(\frac{x}{7}\right)\right)$ as an algebraic expression in x. (7 points)

8) Find $D_w\left[\operatorname{sech}^5\left(e^w\right)\right]$. (7 points)

9) Distances and lengths are measured in meters. (21 points total)



a) Find the **area** of the shaded region R. **Evaluate** your integral completely. Give an **exact** answer in simplest form with appropriate units, and also **approximate** your answer to four significant digits. (8 points)

b) Find the **volume** of the solid generated by revolving the shaded region R about the y-axis. **Evaluate** your integral completely. Give an **exact** answer in simplest form with appropriate units, and also **approximate** your answer to four significant digits. (13 points)