QUIZ ON CHAPTERS 1 AND 2
REVIEW / LIMITS AND CONTINUITY; MATH 150 – SPRING 2017 – KUNIYUKI
105 POINTS TOTAL, BUT 100 POINTS = 100%

Show all work, simplify as appropriate, and use “good form and procedure” (as in class). Box in your final answers! You may not use L'Hôpital's Rule for finding limits. No notes, books, or calculators allowed.

Check one:
Can you easily print files from the class website?

☐ Yes. I can print exam solutions.
☐ No. Give me exam solutions in class.

1) For a), b), and c) below, box in the correct answer. (6 points total; 2 pts. each)
   a) Let \( f(x) = x^4 + \cos(x) \). The function \( f \) is … even odd neither
   b) Let \( g(x) = \sqrt[3]{x} + \sin(x) \). The function \( g \) is … even odd neither
   c) Let \( h(x) = x^5 - x + 1 \). The function \( h \) is … even odd neither

2) Fill in the blanks. Find rules for functions \( f \) and \( g \) so that
\[
(f \circ g)(x) = f(g(x)) = \frac{1}{x^2 + x}
\]
(We are decomposing a composite function.)
\[
g(x) = \underline{\phantom{0}} \quad f(u) = \underline{\phantom{0}}
\]
(Do not let \( f \) or \( g \) be the identity function.) (2 points)

3) Complete the Identities. Fill out the table below so that, for each row, the left side is equivalent to the right side, based on the type of identity (ID) given in the last column. (8 points total; 2 points each)

<table>
<thead>
<tr>
<th>Left Side</th>
<th>Right Side</th>
<th>Type of Identity (ID)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 + \tan^2(u)</td>
<td>\phantom{0}</td>
<td>Pythagorean ID</td>
</tr>
<tr>
<td>\cos(u + v)</td>
<td>\phantom{0}</td>
<td>Sum ID</td>
</tr>
<tr>
<td>\sin(u - v)</td>
<td>\phantom{0}</td>
<td>Difference ID</td>
</tr>
<tr>
<td>\cos^2(u)</td>
<td>\phantom{0}</td>
<td>Power-Reducing ID (PRI)</td>
</tr>
</tbody>
</table>
4) Write any two of the three different versions of the Double-Angle Identity (ID) for \( \cos(2u) \) that were listed in Chapter 1. (4 points)

5) Verify the identity \( \frac{\sin(2x)}{\tan(x)} = 2\cos^2(x) \) using the Chapter 1 identities. (5 points)

6) Fill out the table below. Use interval form (the form using parentheses and/or brackets) except where indicated. You do not have to show work. (6 points)

<table>
<thead>
<tr>
<th>( f(x) )</th>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \cos(x) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tan(x) )</td>
<td>Use set-builder form.</td>
<td></td>
</tr>
<tr>
<td>( \csc(x) )</td>
<td>Use set-builder form.</td>
<td></td>
</tr>
</tbody>
</table>

When evaluating limits, give a real number, \( \infty \), \( -\infty \), or DNE (Does Not Exist). Write \( \infty \) or \( -\infty \) when appropriate. If a limit does not exist, and \( \infty \) and \( -\infty \) are inappropriate, write “DNE.”

7) In parts a) through i), consider the function \( f \), where \( f(x) = \frac{x^2 - 4}{3x^2 - 5x - 2} \), and the graph of \( y = f(x) \) in the usual \( xy \)-plane. (28 points total)

   a) Find \( \lim_{x \to 2} f(x) \). \hspace{1cm} \text{Show all rigorous work, as in class.} \hspace{1cm} (7 points)
b) Find $\lim_{{x \to \infty}} f(x)$. Show all rigorous work, as in class. (5 points)

c) Which of the following is true? Box in one: (2 points)
   i. $f$ is continuous at $x = -2$.
   ii. $f$ has a removable discontinuity at $x = -2$.
   iii. $f$ has a jump discontinuity at $x = -2$.
   iv. $f$ has an infinite discontinuity at $x = -2$.

d) Which of the following is true? Box in one: (2 points)
   i. $f$ is continuous at $x = -\frac{1}{3}$.
   ii. $f$ has a removable discontinuity at $x = -\frac{1}{3}$.
   iii. $f$ has a jump discontinuity at $x = -\frac{1}{3}$.
   iv. $f$ has an infinite discontinuity at $x = -\frac{1}{3}$.

e) Which of the following is true? Box in one: (2 points)
   i. $f$ is continuous at $x = 2$.
   ii. $f$ has a removable discontinuity at $x = 2$.
   iii. $f$ has a jump discontinuity at $x = 2$.
   iv. $f$ has an infinite discontinuity at $x = 2$.

f) What is the horizontal asymptote of the graph of $y = f(x)$? Write its equation. **Answer only on f) is fine.** (3 points)

g) Where is the hole on the graph of $y = f(x)$? Use $(x, y)$ form to write its coordinates. **Answer only on g) is fine.** (3 points)

h) What is the $x$-intercept of the graph of $y = f(x)$? (2 points)

i) What is the $y$-intercept of the graph of $y = f(x)$? (2 points)
8) Let \( f(x) = \frac{3x^4 + x^2 - 1}{4x^{10} - x^5} \). What is the equation of the horizontal asymptote of the graph of \( y = f(x) \) in the usual \( xy \)-plane? (3 points)

9) Find the following limits. **Box in your final answers.** (22 points total)

a) \( \lim_{r \to 11} \frac{\sqrt{r - 2} - 3}{11 - r} \). **Show all work, as in class.** (10 points)

b) \( \lim_{x \to \infty} \frac{\cos(\sqrt[3]{x})}{\sqrt[3]{x}} \). **Show all work, as in class.** (6 points)

c) \( \lim_{x \to 3^+} \frac{x - 6}{x^2 - x - 6} \). **Show all work, as in class.** (6 points)
10) (2 points). True or False: It is possible that for a function $f$, $\lim_{x \to a} f(x)$ exists but $f(a)$ does not (meaning $f(a)$ is undefined). Box in one: True False

11) Write a precise $\epsilon$-$\delta$ definition of $\lim_{x \to a} f(x) = L$, $L \in \mathbb{R}$. Assume $f$ is defined on a punctured neighborhood of $a$. (7 points)

12) Let $f(x) = \begin{cases} 3x - 4, & x \leq 2 \\ \frac{2}{x}, & x > 2 \end{cases}$. (6 points total)
   a) Evaluate $\lim_{x \to 2^+} f(x)$. (2 points)
   b) Evaluate $\lim_{x \to 2^-} f(x)$. (2 points)
   c) Which of the following is true? Box in one: (2 points)
      i. $f$ is continuous at $x = 2$.
      ii. $f$ has a removable discontinuity at $x = 2$.
      iii. $f$ has a jump discontinuity at $x = 2$.
      iv. $f$ has an infinite discontinuity at $x = 2$.

13) Let $g(t) = \frac{1}{\sqrt{t-4}} + \sqrt[3]{t-7}$. What is the domain of $g$? Write your answer in interval form (the form using parentheses and/or brackets). Note: $g$ is continuous on the domain interval(s). (4 points)

14) (2 points). True or False: If $f$ is a polynomial function on $\mathbb{R}$ such that $f(1) = 10$ and $f(7) = 20$, then the equation $f(x) = 15$ has a real solution. Box in one: True False