Math 150 Name: ________________________

QUIZ ON CHAPTER 3

DERIVATIVES; MATH 150 – FALL 2016 – KUNIYUKI
105 POINTS TOTAL, BUT 100 POINTS = 100%

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed. A scientific calculator is allowed.

1) Use the limit definition of the derivative to prove that $D_x\left[\sin(x)\right]=\cos(x)$, as in class. Show all steps! (12 points)

THIS IS THE <u>ONLY</u> PROBLEM ON THIS TEST WHEN YOU WILL NEED TO USE THE LIMIT DEFINITION OF THE DERIVATIVE!

2) A particle moving along a coordinate line has as its position function *s*, where $s(t)=3t+\dfrac{2}{\sqrt[3]{t^2}}$, for $t \geq 1$. Position $s(t)$ is measured in meters, and time *t* is measured in seconds. Find the velocity of the particle at time $t=8$ (seconds). Write an exact answer with correct units. (8 points)

3) If $f(x)=\sqrt{9-x^2}$, is f differentiable on the interval $[-3, 3]$? Box in one: (1 point)

Yes　　　　　　　　No

4) Let $f(x)=x^2+x$. These are linearization problems. (11 points total)

a) Find an equation of the tangent line to the graph of $y=f(x)$ at the point on the graph (in the usual *xy*-plane) where $x=3$. Use any form. (8 points)

b) Use your tangent line from a) to give a linear approximation for the value of $f(2.85)$. Give your answer as a decimal written out to two decimal places. (3 points)

5) Find the indicated derivatives. (21 points total; 7 points each)

a) Find $\frac{d}{dr}\left(\sqrt[7]{r^6}-\frac{5}{r^3}+6\right)$. Simplify. Do not leave negative exponents in your final answer. Your final answer does not need to be a single fraction.

b) Let $f(x)=3x^7\sin\left(x^2\right)$. Find $f'(x)$. Simplify. You do not have to factor your final answer.

c) Find $D_\alpha\left[\csc^3\left(7\alpha+\dfrac{\pi}{4}\right)\right]$. Simplify.

6) Find $D_x\left(\dfrac{7x+1}{\sqrt{3x^2+5}}\right)$. Simplify. Your final answer must have the form:

$\dfrac{\text{an } x \text{ term and a constant term}}{\left(3x^2+5\right)^{\text{some exponent}}}$. (12 points)

7) Prove that $D_x\left[\cot(x)\right] = -\csc^2(x)$ <u>without</u> using the limit definition of the derivative. Among the trigonometric derivatives, you may use only the derivatives of $\sin(x)$ and $\cos(x)$ without proof. Show all work! (8 points)

8) Let $f(x) = \left(x^2 - 4\right)^3$. (10 points total)

   a) Find $f'(x)$.

   b) Find the points on the graph of $y = f(x)$ (in the usual *xy*-plane) at which the tangent line is horizontal.

9) Consider the given equation $6y+4x^5y^2-\cos(y)=12$. Assume that it "determines" an implicit differentiable function f such that $y=f(x)$. Find $\frac{dy}{dx}$ (you may use the y' notation, instead). Use implicit differentiation, as in class. (11 points)

10) Air is being pumped into a spherical balloon at the rate of 2.3 cubic feet per minute. The balloon maintains a spherical shape throughout. At what rate is the radius of the balloon changing when the radius is 30 **inches** in length?

• In your final answer, give the appropriate units, and round off your answer as a decimal to four significant digits. (Round off intermediate results to at least four significant digits.)

• Hint 1: If you forgot the "key formula" here, you can buy it from me for 2 points. You can't get negative points for this problem.

• Hint 2: One foot is equivalent to 12 inches.
(11 points)