1) Use the limit definition of the derivative to prove that $D_x \left[ \sin(x) \right] = \cos(x)$, as in class. Show all steps! (12 points)

**THIS IS THE ONLY PROBLEM ON THIS TEST WHEN YOU WILL NEED TO USE THE LIMIT DEFINITION OF THE DERIVATIVE!**

2) A particle moving along a coordinate line has as its position function $s$, where $s(t) = 3t + \frac{2}{\sqrt{t^2}}$, for $t \geq 1$. Position $s(t)$ is measured in meters, and time $t$ is measured in seconds. Find the velocity of the particle at time $t = 8$ (seconds). Write an exact answer with correct units. (8 points)
3) If \( f(x) = \sqrt{9 - x^2} \), is \( f \) differentiable on the interval \([-3, 3]\)? Box in one: (1 point)

Yes
No

4) Let \( f(x) = x^2 + x \). These are linearization problems. (11 points total)
   a) Find an equation of the tangent line to the graph of \( y = f(x) \) at the point on the graph (in the usual \( xy \)-plane) where \( x = 3 \). Use any form. (8 points)

b) Use your tangent line from a) to give a linear approximation for the value of \( f(2.85) \). Give your answer as a decimal written out to two decimal places. (3 points)

5) Find the indicated derivatives. (21 points total; 7 points each)
   a) Find \( \frac{d}{dr} \left( \sqrt[3]{r^6} - \frac{5}{r^3} + 6 \right) \). Simplify. Do not leave negative exponents in your final answer. Your final answer does not need to be a single fraction.

b) Let \( f(x) = 3x^7 \sin(x^2) \). Find \( f'(x) \). Simplify. You do not have to factor your final answer.
c) Find $D_\alpha \left[ \csc^3 \left( 7\alpha + \frac{\pi}{4} \right) \right]$. Simplify.

6) Find $D_x \left( \frac{7x + 1}{\sqrt{3x^2 + 5}} \right)$. Simplify. Your final answer must have the form: 

\[
\frac{\text{an x term and a constant term}}{(3x^2 + 5)^{\text{some exponent}}}. \quad (12 \text{ points})
\]
7) Prove that \( D_x[\cot(x)] = -\csc^2(x) \) without using the limit definition of the derivative. Among the trigonometric derivatives, you may use only the derivatives of \( \sin(x) \) and \( \cos(x) \) without proof. Show all work! (8 points)

8) Let \( f(x) = (x^2 - 4)^3 \). (10 points total)
   a) Find \( f'(x) \).
   b) Find the points on the graph of \( y = f(x) \) (in the usual xy-plane) at which the tangent line is horizontal.
9) Consider the given equation \( 6y + 4x^5y^2 - \cos(y) = 12 \). Assume that it “determines” an implicit differentiable function \( f \) such that \( y = f(x) \).

Find \( \frac{dy}{dx} \) (you may use the \( y' \) notation, instead). Use implicit differentiation, as in class. (11 points)

10) Air is being pumped into a spherical balloon at the rate of 2.3 cubic feet per minute. The balloon maintains a spherical shape throughout. At what rate is the radius of the balloon changing when the radius is 30 inches in length?

• In your final answer, give the appropriate units, and round off your answer as a decimal to four significant digits. (Round off intermediate results to at least four significant digits.)

• Hint 1: If you forgot the “key formula” here, you can buy it from me for 2 points. You can’t get negative points for this problem.

• Hint 2: One foot is equivalent to 12 inches.

(11 points)