Math 150 Name: ________________________

QUIZ ON CHAPTER 5

INTEGRALS; MATH 150 – FALL 2016 – KUNIYUKI
105 POINTS TOTAL, BUT 100 POINTS = 100%

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed. A scientific calculator is allowed.

Final answers must not consist of any negative exponents or compound fractions.

1) Evaluate the following integrals. Simplify as appropriate. (42 points total)

a) $\int \frac{5r^2 - 7r + \sqrt{r}}{r}\, dr$ (9 points)

b) $\int \cot(\theta) \sec(\theta) \csc(\theta)\, d\theta$ (7 points)

c) $\int_0^8 \left| x-3 \right| dx$ (7 points)

(You may use the Fundamental Theorem of Calculus or a geometric argument.)

d) $\int \frac{\csc(x)\cot(x)}{\left[1+\csc(x)\right]^3}\, dx$ (7 points)

e) $\int_1^4 \frac{x}{\sqrt{3x^2+1}}\,dx$ (12 points)

Give an exact, simplified fraction as your answer.

2) Solve the second-order differential equation $\frac{d^2y}{dx^2} = \cos(2x)$ subject to the following initial conditions: $\frac{dy}{dx} = \frac{5}{4}$ when $x = \frac{\pi}{12}$, and $y = 1$ when $x = 0$. (14 points)

3) Assume that f is an everywhere continuous function on $\mathbb{R}$ such that $\int_2^5 f(x)\,dx = 20$. Simplify and evaluate: $\int_5^8 f(x)\,dx + \int_8^2 f(x)\,dx$. (5 points)

4) Assume that f is an everywhere continuous **even** function on $\mathbb{R}$ such that $\int_0^{10} f(x)\,dx = 30$. Evaluate $\int_{-10}^{10} 7f(x)\,dx$. (4 points)

5) Let $f(x) = x^4 + 3$. (12 points total)

   a) Find f_{av}, the average value of f on the interval $[-1, 2]$.
   Write your answer as an exact, simplified fraction. (10 points)

   b) True or False: There exists a real number z in the interval $(-1, 2)$ such that $f(z) = f_{av}$, where f_{av} is the [correct] answer to part a). Box in one: (Don't worry about the issue of open vs. closed intervals.) (2 points)

   True   False

6) Simplify $D_x\left(\int_5^x \frac{1}{t^2+1}\,dt\right)$. (3 points)

7) We will approximate $\int_0^1 \tan(x)\,dx$ using two different methods.
For part a) and part b), use a **regular** partition with $n = 4$ subintervals.
Round off intermediate calculations to at least five significant digits, and round off your final answers to four significant digits. Show all work! Remember that x is to be measured in radians, as opposed to degrees. (15 points total)

a) Approximate $\int_0^1 \tan(x)\,dx$ by using a Left-hand Riemann Approximation (LRA).

b) Approximate $\int_0^1 \tan(x)\,dx$ by using the Trapezoidal Rule.
Hint: The Trapezoidal Rule is given by:

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left[f(x_0)+2f(x_1)+2f(x_2)+\cdots+2f(x_{n-1})+f(x_n)\right], \text{ or}$$

$$\approx \frac{1}{2}\Delta x\left[f(x_0)+2f(x_1)+2f(x_2)+\cdots+2f(x_{n-1})+f(x_n)\right]$$

8) Evaluate $\int \frac{x}{(x-10)^{50}}\,dx$. Do not leave negative exponents in your final answer.

Hint: Remember a trick discussed in class. (10 points)