1) Evaluate the following integrals. Simplify as appropriate. (47 points total)

a) \[ \int \frac{(w^2 + 4)^2}{w^2} \, dw \]  \hspace{1cm} (9 points)

b) \[ \int \frac{1 + \tan^2(\theta) \tan(\theta)}{\sec(\theta)} \, d\theta \]  \hspace{1cm} (6 points)

c) \[ \int 3 \sec^2(x) \tan^7(x) \, dx \]  \hspace{1cm} (7 points)
d) \[ \int \frac{(2 + \sqrt{x})^6}{\sqrt{x}} \, dx \] (8 points)

e) \[ \int_0^3 \sqrt{9-x^2} \, dx \] (5 points)
(Hint: Do not use the Fundamental Theorem of Calculus.)

f) \[ \int_0^2 \frac{x^2}{\sqrt{2x^3 + 9}} \, dx \] (12 points)
Give an exact, simplified fraction as your answer.
2) An astronaut crawls to the edge of a cliff on planet Dork. The edge lies 33 feet above a lake. The astronaut throws down a rock at 30 feet per second.

The acceleration function for the rock is given by \( a(t) = -6 \frac{\text{ft}}{\text{sec}^2} \), which is the [signed] gravitational constant for Dork. The variable \( t \) represents time in seconds after the rock was thrown. Find the height function \( s(t) \) for the height of the rock above the lake. \( s(t) \) is measured in feet. [Note: Your \( s(t) \) rule will only be relevant between the time the rock is thrown and the time the rock hits the lake.] Show all work, as in class. (9 points)
3) Assume that \( f \) is an everywhere continuous function on \( \mathbb{R} \) such that
\[
\int_{10}^{20} f(x) \, dx = 100.
\]
evaluate:
\[
\int_{10}^{20} \left[ 3f(x) - 1 \right] \, dx .
\]
(5 points)

4) Evaluate:
\[
\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \tan(x) \, dx .
\]
Answer only is fine. (2 points)

5) Simplify:
\[
D_x \left( \int_{\pi}^{x} \sin(t^2) \, dt \right).
\]
(2 points)

6) For parts a), b), c), and d), let \( f(x) = x^4 \). (30 points total)

a) Approximate
\[
\int_{2}^{4} x^4 \, dx
\]
by using a Right-hand Riemann Approximation (RRA) based on the partition \( \{2.0, 2.7, 3.0, 3.6, 4.0\} \). Round off calculations to at least five significant digits. (10 points)
b) Approximate \( \int_2^4 x^4 \, dx \) by using the Trapezoidal Rule. Use a regular partition with \( n = 4 \) subintervals. Round off calculations to at least five significant digits. (12 points)

Hint: The Trapezoidal Rule is given by:
\[
\int_a^b f(x) \, dx \approx \frac{b-a}{2n} \left[ f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n) \right], \text{ or } \\
\approx \frac{1}{2} \Delta x \left[ f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n) \right]
\]

c) Find the exact value of \( \int_2^4 x^4 \, dx \) by applying the Fundamental Theorem of Calculus. (5 points)

d) Use part c) to find \( f_{av} \), the average value of \( f \) on the \( x \)-interval \([2, 4]\), where \( f(x) = x^4 \). (3 points)
7) Evaluate \( \int x(x + 5)^{200} \, dx \). Hint: Remember a trick discussed in class. (10 points)